HEDGING AGAINST COMMODITY PRICE INFLATION: A SECURITY MARKET APPROACH

BY

GERALD DOUGLAS GAY

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Gerald Douglas Gay

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Investors often hedge against price inflation in a particular commodity by taking a position in a futures contract, an action that protects the investor from any price fluctuations in the commodity during the period before delivery. This dissertation develops portfolios composed of default-free bills of various maturities and shares of common stock which allow an investor to hedge against commodity price inflation, without actually entering the futures markets. This hedging alternative can be of use to investors who wish to hedge against price inflation in commodities for which organized futures trading does not exist.

This study develops hedges by purchasing existing security market instruments, using a previously suggested technique. These hedge portfolios, or "quasi-futures" contracts, for a particular commodity are constructed such that they are not only highly correlated with the commodity, but also have the same price and risk properties. The portfolios and the commodities are shown to have the same expected price at delivery, and the same covariances with the stock market, all other commodities, and long-term bills.

The hedging portfolios developed for ten various commodities over the period 1965-1976 are shown to be unbiased substitutes for the commodities themselves. The techniques explored in this dissertation allow investors to hedge against price inflation in any commodity. Consequently, the existence of these "quasi-futures" contracts has important implications for the necessity of futures markets and for the development of a multi-period capital asset pricing model. If markets are perfect and securities are traded costlessly, then futures markets do not provide an investor with a service that is not already available in the existing markets for common stocks and Treasury bills. Additionally, since these "quasi-futures" contracts can be created with existing securities, a pricing model, based on a multi-period economy which allows for shifts in commodity prices and the term structure of interest rates, may well lead to a more complete and realistic view of capital market equilibrium. Finally, the "quasi-futures" contracts are shown to require frequent rebalancing. Thus, futures markets may be economically valuable since they allow investors to hedge more efficiently in terms of transaction costs.

CHAPTER I
INTRODUCTION

I. A Overview

Investors often hedge against price inflation in a particular commodity by taking a position in a futures contract, an action that protects the investor from any price fluctuations in the commodity during the period before delivery. This dissertation investigates whether portfolios composed of default-free bills of various maturities and shares of common stock allow an investor to hedge against commodity price inflation, without actually entering the futures markets. This hedging alternative will be especially appealing to investors who wish to hedge against price inflation in commodities for which organized futures trading does not exist.

This study develops hedges by purchasing existing security market instruments, using a technique suggested by Long (1974). These hedge portfolios, or "quasi-futures" contracts,[1] for a particular commodity will not only be highly correlated with the commodity, but will have all the same price and risk properties. The portfolios and the commodities will have the same expected price at delivery, and the same covariances with the stock market, all other commodities, and long-term bills.

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[1]These portfolios are not true futures contracts for two reasons; (1) they require a positive net investment, and (2) they involve taking a current position in the markets for common stocks and Treasury bills. However, these portfolios will be called "quasi-futures" contracts to be consistent with the terminology of Long.

The hedging portfolios developed for ten various commodities over the period 1965-1976 will be shown to be unbiased substitutes for the commodities themselves. Consequently, their existence has important implications for the necessity of futures markets and for the development of a multi-period capital asset pricing model. If markets are perfect and securities are traded costlessly, then futures markets will not provide an investor with a service that is not already available in the existing markets for common stocks and Treasury bills. Additionally, if these "quasi-futures" contracts can be created with existing securities, Long's pricing model, based on a multi-period economy which allows for shifts in commodity prices and the term structure of interest rates, may well lead to a more complete and realistic view of capital market equilibrium.

I. B Previous Work in Commodity Pricing

Futures markets integrate the actions of two types of investors who are typically described as being either hedgers or speculators. In order to avoid the risks of price fluctuations in the spot commodity, a hedger will initiate a futures market transaction. Thus an investor who desires a particular commodity for future consumption or production purposes will purchase a claim for future delivery of the specified commodity. This is a riskless claim for consumption of the commodity even though the future spot price of the commodity is highly uncertain. On the other side of the market, the speculators underwrite the risks of price fluctuation in the spot commodity in hopes of receiving some compensation.

Keynes (1930) first analyzed futures markets as constituting an insurance mechanism. According to Keynes, hedgers pay a significant premium to the speculators for underwriting the risks of price fluctuation in a commodity. Hardy (1940) has argued to the contrary, that futures

markets are a socially acceptable form of gambling whereby speculators may actually be willing to pay for this opportunity to gamble. Thus the premium they receive should be zero or possibly negative.

Attempts have been made to analyze the returns to holders of futures contracts for the purpose of interpreting the actual need for futures markets. Dusak (1973) presents a portfolio approach and argues that futures markets are no different in principle from the markets for any other risky portfolio assets. Since all assets are candidates for inclusion in an investor's portfolio, the return on any risky asset should be governed by the asset's contribution to the risk of a large and well-diversified portfolio of assets. In her paper Dusak investigates the risk-return relationship to holders of futures contracts in a capital asset pricing framework.

The purchase of a futures contract is like buying a capital asset on credit since the buyer has no capital of his own invested. The margin paid by an investor is merely a good-faith deposit to acknowledge a later commitment to the contract and thus cannot be treated as a capital investment. Thus, in order to approximate the risk premium earned on the spot commodity, Dusak uses the percentage change in the futures price over a given interval. Her results indicate that the returns and systematic portfolio risk are both close to zero for each of the commodity futures studied (wheat, corn, and soybeans) despite the fact that each commodity had a large price variability during the sample period of May 1952 through November 1967. These findings contradict the Keynesian theory which says an investor in commodity contracts should earn a substantial positive return. In addition, the findings only partially support the Hardy gambling casino theory (which predicts a mean return of zero) since the systematic risk was close to zero.

Black (1976) also discusses the behavior of futures prices in a model of capital market equilibrium and states that the returns on commodity holdings should obey the capital asset pricing model like any other asset. He develops an expression which states that the expected change in the futures price is proportional to the "dollar beta" of the futures price. If the change in the futures price is uncorrelated with the return on the market portfolio, thus producing a zero beta for the futures price, then one would expect a zero change in the futures price. Had Black empirically tested his equations, he would have expected the same zero returns to holders of futures contracts that Dusak found, since in Dusak's paper covariances with the stock market were close to zero for wheat, corn, and soybean futures.

Black also states that since commodity holdings appear to be priced like other assets, then investors who own commodities should be able to diversify away any unsystematic risk. One way this may be done is through futures markets. However, since the majority of commodity holdings is by corporations, the risk may be passed on to shareholders who should hold well-diversified portfolios. This implies that futures markets do not have a unique role in the allocation of commodity price risk since corporations can do a more efficient job, especially in the cases where organized futures markets are nonexistent for many commodities.

Finally, on the need for futures markets, Black refers to the information content of futures markets. Black (1976, p. 176) states "I believe that futures markets exist because in some situations they provide an inexpensive way to transfer risk, and because many people both in the business and out like to gamble on commodity prices. Neither of these count as a major benefit to society. The big benefit from futures markets

is the side effect: the fact that participants in the futures markets can make production, storage, and processing decisions by looking at the pattern of futures prices even if they don't take positions in that market."

Stoll (1978) also maintains that traditional explanations of hedging fail to take proper account of the available risk spreading opportunities in the capital market as a whole as opposed to futures markets alone. His rationale for hedging is the inability or reluctance of individuals such as farmers or privately held firms to trade ownership claims on certain assets or production techniques with which they are endowed.

Stoll describes two types of risk associated with a commodity: (1) price risk arising from future supply and demand uncertainties, and (2) the risk due to uncertainties in storage costs. Futures markets only allow the price risk to be passed on, but both the price and storage risks could be passed on in the stock market if the process has shares traded.

Stoll develops a model of futures prices in a capital market equilibrium framework in which there are non-tradeable assets. He relates the expected dollar return on a futures contract to the market price of risk and the risk of the futures contract which is measured by the covariance of the commodity return with the return of all other assets. However, even if this systematic market risk is zero as Dusak found, the expected return on futures contracts may not be zero depending on the size of the market value of commodities relative to the market value of all shares of stock. He concludes that in a world of perfect capital markets there is no need for hedging if futures contracts and shares in the production or storage process are traded at no cost.

It appears there is beginning to be some success in developing a pricing framework to analyze futures markets. However, there is a lack of any empirical testing of the models in the literature, and even less to substantiate the different authors' explanations for the actual need of futures markets. In their papers, neither Black nor Stoll empirically tests his model. Moreover, Stoll's pricing model for futures contracts, to be properly tested, requires pricing information on non-tradeable assets. Dusak did construct and perform some tests, but limited her investigation to three commodities.

Correct development of an equilibrium pricing model for futures contracts and subsequent empirical analysis of the returns to holders of futures contracts would not be adequate criteria for justifying the need for futures markets as other authors have claimed. The necessity of futures markets depends on the contribution of futures markets to the completeness of markets. That is, are the services and investment opportunities provided by futures markets unique in that they are not available elsewhere?

I. C General Outline

This dissertation investigates whether there are assets in existing stock and bill markets which can be combined in such a way that the resulting portfolio will have the same price and risk characteristics as does a particular commodity. A stock-bill portfolio of this type which serves as a substitute to holding a good itself will be referred to as a "quasi-futures" contract. These "quasi-futures" contracts for a particular commodity should have (1) the same "with-dividend" value at delivery as the spot commodity and (2) the same covariance with the stock market, all other goods, and all long-term bills as does the good itself.

The concept of a "quasi-futures" contract was first presented by Long (1974) in his development of a multi-period capital asset pricing model. The next chapter presents Long's theoretical discussion for the development of his model from which the concept of a "quasi-futures" contract is taken. The economy in Long's model is very realistic in that there is a stock market, a market for default-free bills of different maturities, and many consumption goods whose future prices are uncertain. Long relates the price of an asset to not only the systematic market risk, but also to the risk due to changing consumption opportunities (inflation risk) and changing investment opportunities (interest rate risk).

Chapter III presents the methodology for creating "quasi-futures" contracts. The steps needed to attain the correct expected price and covariance properties of the hedge portfolios are described in detail.

Chapter IV describes the actual formation of "quasi-futures" contracts for ten different commodities using asset data from the period January 1961 through March 1976. Selection of the various commodity, bill, and stock data to be used, as well as the formation of a market index, will be discussed. The econometric difficulties encountered in creating the asset weights for the hedging porfolios are also presented. Finally, the composition of the "quasi-futures" contracts is examined with regards to the relative weights of the various stocks and bills contained in each period's hedge. These findings are then compared with those of Fama and Schwert (1977a), who investigate the success of various assets as hedging devices against the different components of the inflation rate.

Chapter V outlines the various tests that are conducted in order to verify that the "quasi-futures" contracts constructed are indeed true substitutes for the goods themselves. Hypotheses are tested to determine whether each hedging portfolio and good had the same subsequently observed prices and covariances with the stock market, all other goods, and all long-term bills. Interpretation of these results and the implications for the necessity of futures markets is discussed. In addition, the sensitivity of the results to the Nixon wage and price controls is tested.

Since the "quasi-futures" contracts are one month hedges which require extensive and costly rebalancing, the tests of hypotheses are repeated for the cases where the portfolios are rebalanced only at quarterly and semiannual intervals. The results from these tests indicate that futures markets provide a less expensive means for hedging against commodity price inflation versus the use of "quasi-futures" contracts.

The final chapter presents the conclusions of the dissertation and draws the practical implications of the research for hedging. In addition, potential extensions of the dissertation are discussed.

CHAPTER II
A LITERATURE REVIEW OF MULTI-PERIOD PRICING MODELS

The capital asset pricing model as developed by Sharpe (1964) and Lintner (1965a,b) abstracts from reality by assuming consumers act to maximize the expected value of a utility function whose only arguments are consumption at time 0 and nominal wealth at time 1. To make this assumption consistent with consumer maximization of the expected utility of a lifetime consumption stream, it becomes necessary to assume future prices of consumption goods are known with certainty and no unpredictable changes occur in the investment opportunities over time. Uncertainty in the rate of commodity price inflation is thereby eliminated. In addition, the capital market cannot contain assets whose future rates of return over a time interval may depend on unanticipated events in the interim. Thus, an economy with a bill market is not allowed. However, commodity price inflation and the term structure of interest rates may play an important role in the determination of equilibrium asset prices.

Roll (1973) recognizes that inflation of commodity prices has been almost completely neglected in the literature of asset pricing. He develops a capital asset pricing model that includes the risk of currency inflation and attempts to show how assets and commodities acquire equilibrium prices in competitive markets. Roll points out that the simple Fisherian equation will not correctly specify the relation between nominal interest rates and expected rates of commodity price inflation. His analysis indicates that the nominal expected rate of return on assets will depend on the covariance between nominal asset returns and the rate

of inflation. Roll's economy, however, has only one future date and thus the term structure issue is ignored.

Merton (1972) presents a continuous time analysis of the demand for assets in an economy where both consumption good prices and the investment opportunity set are allowed to vary randomly over time. He derives a formula that expresses the consumer's demand for risky assets in term of parameters describing his consumption preferences and parameters of the stochastic process governing commodity and asset prices. Like Roll, however, Merton's paper fails to provide testable propositions in the form of precise relations between well defined and readily measurable variables.

II. A Long's Multi-Period Pricing Model

Long (1974) provides a very realistic view of capital market equilibrium, by presenting a multi-period discrete time analysis which deals directly with uncertainty in future commodity prices and future investment opportunities from which he develops empirically testable price formulas for common stock and long-term bills. These formulas are developed such that the relation between equilibrium prices and parameters describing comsumer preferences can be directly analyzed. The price of an individual asset can be interpreted in terms of its marginal contributions to portfolio characteristics that concern investors such as mean, variance, and covariance.

The economy in Long's model contains three markets: a stock market where the shares of N firms are traded, a consumption goods market containing K non-storable commodities available to consumers, and a market containing default-free bills for any maturity date up to and including time T. For each of these markets, the following assumptions are made:

(1) markets are perfect in the sense that all items are infinitely divisible, there are no transaction costs, and all traders acts as price takers; (2) on any date all traders have free and equal access to all information which is relevant to assessing the subjective joint probability distributions of prices which prevail on subsequent trading dates and, furthermore, all consumers form identical expectations regarding prices to be realized on subsequent dates; (3) markets are only open on trading dates which are equally spaced in time; (4) all production in the economy is accomplished by firms; and (5) the only inputs to production supplied directly by consumers are capital funds supplied by the purchase of the firm's shares and bills.

Next, a brief development of the pricing formulas is presented. The notation that will be used is given as follows:

Π_{kt} = the price per unit of good k at time t;

C_{ikt} = the quantity of good k purchased by consumer i at time t;

P_{jt} = the ex-dividend price per share of stock in firm j at time t;

D_{jt} = the dividend per share paid at time t to shareholders in firm j;

V_{jt} = the "with-dividend" price per share of stock in firm j at time t;

X_{ijt} = the number of shares in firm j that consumer i chooses at time t to hold during the (t+1)st period;

B_{tm} = the price at time t of a bill which matures and pays one dollar at time m, $(0 \le t < m \le T)$;

Y_{itm} = the number of bills maturing at time m that consumer i chooses at time t to hold during the (t+1)st period.

A vector of prices or quantities will be referred to by omitting the subscript which indexes the elements of the vector. For example, the bundle of goods consumed at time t by consumer i is denoted by $C_{it} \equiv (C_{i1t}, C_{i2t}, \ldots, C_{iKt})'$.

Consumer i is assumed to act at time 0 as if he is maximizing the expected value of a utility function of the form $U^i(C_{i0}, C_{i1}, \ldots, C_{iT})$ where U^i is monotonically increasing, strictly concave, continuous, and twice differentiable with respect to its argument. The maximization is done subject to a set of budget constraints which will be given later.

Fama (1970b) shows how this problem can be recharacterized as one in which consumer i acts at time 0 as if he is maximizing the expected value of a semi-indirect utility function of the form $F^i(C_{i0}, w_{i1}, \phi_1)$. F^i is defined to be the maximum attainable value of $E(U^i)$ given that C_{i0} is the bundle of consumption goods chosen at time 0, w_{i1} is the realized value of time 1 nominal wealth, and that ϕ_1 is the realized value of the vector of time 1 prices and dividends $\tilde{\phi}_1 \equiv (\tilde{\Pi}, \tilde{P}_1, \tilde{D}_1, \tilde{B}_1)$.

The variables w_{i1} and ϕ_1 serve to summarize the opportunity set the consumer will face at time 1. However, it is not necessary to have all of the data specified by w_{i1} and ϕ_1 in addition to those items known to the consumer at time 0 in order to fully specify the opportunity set seen at time 1. Empirical evidence (see Fama, 1970a) suggests that observed stock prices and dividends do not convey new information about the distribution of rates of return to be earned over subsequent periods that is not already provided by observation of good and bill prices alone. P_1 and D_1 may then be eliminated as arguments of the function F^i and, therefore, we are left to maximize $E\,[F^i\,(C_{10}, \tilde{w}_{i1}, \tilde{\Pi}_1, \tilde{B}_1)]$ with respect to (C_{i0}, X_{i0}, Y_{i0}).

In order to simplify the problem further, the consumer's subjective joint probability distribution on $\tilde{\Pi}_1$, $\tilde{B}_1$, and $\tilde{w}_1$ is assumed to be multivariate normal at time 0. The expected value of F^i then becomes a function of C_{i0} and the parameters which identify the particular multivariate normal distribution of $(\tilde{w}_{i1}, \tilde{\Pi}_1, \tilde{B}_1)$. The only parameters of the distribution that are affected by the consumer's portfolio choice are 1) the expected value of $\tilde{w}_{i1}$, 2) the variance of $\tilde{w}_{i1}$, 3) the covariance of $\tilde{w}_{i1}$ with each of the time 1 consumption good prices $\{\tilde{\Pi}_{k1}, k=1, \ldots, K\}$, and 4) the covariance of $\tilde{w}_{i1}$ with each of the time 1 bill prices $\{\tilde{B}_{1m}, m=2, \ldots, T\}$. Thus there exists a function G^i that can now be used in place of the expected value of F^i as the objective function such that

$$G^i(C_{i0}, e_i, v_i, H_i, J_i) = E[F^i(C_{i0}, \tilde{w}_{i1}, \tilde{\Pi}_1, \tilde{B}_1)]$$

where

$$e_i \equiv E(\tilde{w}_{i1});$$

$$v_i \equiv \text{var}(\tilde{w}_{i1});$$

$$H_i \equiv (H_{i1}, \ldots, H_{ik}, \ldots, H_{iK})' \text{ with } H_{ik} = \text{cov}(\tilde{w}_{i1}, \tilde{\Pi}_{k1});$$

and

$$J_i \equiv (J_{i2}, \ldots, J_{im}, \ldots, J_{iT})' \text{ with } J_{im} = \text{cov}(\tilde{w}_{i1}, \tilde{B}_{1m}).$$

Consumer i's current consumption-investment decision problem at time 0 will then be to maximize G^i subject to the constraints:

$$\sum_{k=1}^{K} C_{ik0}\Pi_{k0} + \sum_{j=1}^{N} X_{ij0}P_{j0} + \sum_{m=1}^{T} Y_{i0m}B_{0m} = w_{i0};$$

$$C_{ik0} \geq 0, \qquad k = 1, \ldots, K;$$

and

$$\tilde{W}_{i1} \equiv \sum_{j=1}^{N} X_{ij0}\tilde{V}_{j1} + Y_{i01} + \sum_{m=2}^{T} Y_{i0m}\tilde{B}_{1m}.$$

By solving the above maximization problem it can be shown that at equilibrium the resulting pricing equation for stocks and bonds is[1]

$$P_0 = B_{01}[\bar{V}_1 + 2v^{-1}\Theta_{ev}(\underline{\sigma}_M + \Gamma b) + \phi\underline{\Theta}_{eH} + \Gamma\underline{\Theta}_{eJ}] \qquad \text{(II-1)}$$

$$B_0 = B_{01}[\bar{B}_1 + 2v^{-1}\Theta_{ev}(\underline{S}_M + \Sigma_B b) + \Psi\underline{\Theta}_{eH} + \Sigma\underline{\Theta}_{eJ}] \qquad \text{(II-2)}$$

where

v = the number of consumers,

Θ_{ev} = a weighted average of individual consumers' marginal rates of substitution (MRSs) of expected nominal wealth for variance in nominal wealth,

$\underline{\Theta}_{eH}$ = a vector of weighted averages of individual consumers' MRSs of expected nominal wealth for covariance between nominal wealth and the price at time 1 of each good k,

$\underline{\Theta}_{eJ}$ = a vector of weighted averages of individual consumers' MRSs of expected nominal wealth for covariance between nominal wealth and the price at time 1 of each bill m,

b = a vector of the supplies of long-term bills,

and

$\underline{\sigma}_m$, Φ, Γ, $\underline{S}_m$, Ψ, and Σ_B are vectors of covariances whose elements are listed in Figure III-1 on page 21.

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[1]See Long, Appendix A, for the derivation of the first-order conditions and the resulting price formulas.

It is possible to give economic interpretations to each of the above terms. B_{01}, the price of a risk-free one period bill, equals the MRS of current nominal wealth (w_{i0}) for expected nominal wealth at time 1 (e_i). $\bar{V}_1$ is the marginal contribution per share of stock to the consumer's expected nominal wealth at time 1. The term $2v^{-1}\theta_{ev}(\underline{\sigma}_m + \Gamma b)$ measures the "expected wealth equivalent" to the average consumer of the marginal nominal risk of a share of stock. The term $\phi\underline{\theta}_{eH}$ measures the "expected wealth equivalent" of the marginal contribution per share of stock to covariance between consumer's time 1 wealth ($\tilde{w}_{i1}$) and the price of each consumption good k, ($\tilde{\Pi}_{k1}$). Similarly, $\Gamma\underline{\theta}_{eJ}$ measures the "expected wealth equivalent" of the marginal contribution per share of stock to covariance between $\tilde{w}_{i1}$ and the price at time 1 of bills maturing at time m, ($\tilde{B}_{1m}$).

The price formula for long term bills (II-2) is interpreted in the same way as the stock price formula (II-1) with the words "bills maturing at time m" substituted for "share of stock."

Using historic data, estimations can be formed of the expected values and covariances in equations (II-1) and (II-2). To solve for the unobservable θ's, $\{\theta_{ev}, \theta_{eHk}, \theta_{eJm};\ k = 1, \ldots, K;\ m = 2, \ldots, T\}$ in the formulae, there is a restriction for a non-trivial solution that the number of stocks and long-term bills for which there are pricing formulae must equal or exceed the number of unobservable θ's. In other words, it is required tnat $N + (T - 1) \geq 1 + K + (T - 1)$, or equivalently, that $N \geq K + 1$. If not, a solution will exist for any set of current prices and the model would not have any empirical content.

By solving for the unobservable θ's, it is possible to derive the following reduced form pricing model[2] which not only prices the systematic market risk of a share of stock, but also the inflation risk from changing good prices and the interest rate risk from changes in the term structure:

$$P_{j0} = B_{01}[\bar{V}_{j1} - \beta_{jM}(\bar{V}_{M1} - B_{01}^{-1}P_{M0}) - \sum_{k=1}^{K} \xi_{jk}(\bar{\Pi}_{k1} - B_{01}^{-1}F_{k0})$$

$$-\sum_{m=2}^{T} \delta_{jm}(\bar{B}_{1m} - B_{01}^{-1}B_{0m})], \qquad j = 1, \ldots, N. \qquad \text{(II-3)}$$

In the above equation $\bar{V}_{M1}$ is the mean of the "with-dividend" value of the stock market portfolio at time 1 and P_{M0} is its current ex-dividend price. The assumption that $(\tilde{V}_{M1}, \tilde{\Pi}_1, \tilde{B}_1)$ is multivariate normal guarantees the existence of linear regressions within this set of random variables. The symbols β_{jM}, $\{\xi_{jk}, k = 1, \ldots, K\}$, and $\{\delta_{jm}, m = 2, \ldots, T\}$ are, respectively, the coefficients of $\tilde{V}_{M1}$, $\{\tilde{\Pi}_{k1}, k = 1, \ldots, K\}$ and $\{\tilde{B}_{1m}, m = 2, \ldots, T\}$ in the following multiple regression:

$$\tilde{V}_{j1} = \alpha_j + \beta_{jM}\tilde{V}_{M1} + \sum_{k=1}^{K} \xi_{jk}\tilde{\Pi}_{k1} + \sum_{m=2}^{T} \delta_{jm}\tilde{B}_{1m} + \tilde{\varepsilon}_j, \; j = 1, \ldots, N. \qquad \text{(II-4)}$$

In the above regression, $\tilde{V}_{M1} \equiv \sum_{j=1}^{N} \tilde{V}_{j1}$, the "with-dividend" value at time 1 of the stock market portfolio. Also, $E(\tilde{\varepsilon}_j) = 0$ and $\tilde{\varepsilon}_j$ is independent of $(\tilde{V}_{M1}, \tilde{\Pi}_1, \tilde{B}_1)$. Finally, F_{k0} is the current ex-dividend price of any stock-bill portfolio whose "with-dividend" value at time 1 has a mean equal to $\tilde{\Pi}_{k1}$ and has the same covariance with the elements of $(\tilde{V}_{M1}, \tilde{\Pi}_1, \tilde{B}_1)$ as does $\tilde{\Pi}_{k1}$. This stock-bill portfolio is referred to as a "quasi-futures" contract for good k.

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[2]See Long, Appendix B.

II. B A Test of the Long Multi-Period CAPM

Gouldey (1977) presents a test of the Long model. The purpose of Gouldey's study was to test the implications of the Long model, which suggested that investors are concerned with three types of risk when making investment decisions: the traditional systematic market risk, the risk due to a stochastic consumption opportunities set, and the risk due to stochastically changing investment opportunities.

Whereas the traditional mean-variance CAPM relates the return on a security to only its systematic market risk, the Long CAPM relates the return on a security to 1) the systematic risk of the security in the stock market, 2) the risk of the security due to expected price changes in each commodity k $(k = 1, \ldots, K)$, and 3) the risk of the security with respect to a bill of maturity m $(m = 2, \ldots, T)$, due to anticipated shifts in the yield curve. Since there are thus $K + T$ types of risk, each security can be represented by a point in $(K + T + 1)$ space. Thus, a natural generalization of the security market line (SML) associated with the single-period CAPM is the security market hyperplane (SMH) on which each security in equilibrium must lie. The SMH will intersect the expected return axis at the riskless rate just as the SML does in the two-dimensional case.

Gouldey first tests the Long model using cross-sectional methods across individual securities over the period January 1953 through July 1971. He selected 1953 as a starting date since earlier price indices compiled by the Bureau of Labor Statistics (BLS) are inaccurate and misleading due to poor sampling techniques. Also, before 1951, the Fed pegged the interest rates on Treasury bills, thus not allowing Treasury bill rates to adjust to anticipated variation in inflation rates.

The tests ended in July 1971 since wage and price controls were imposed in August 1971 which caused the various price indices not to reflect the true cost of consumption goods.

Gouldey's data consisted of monthly returns on common stock taken from the monthly returns file of the Center for Research in Security Prices (CRSP) at the University of Chicago. The Standard and Poors Combined Index was used to compute a proxy for the return on the market portfolio. Price indices of the different groupings used by the Bureau of Labor Statistics in compiling the Consumer Price Index (CPI) were used for computing monthly returns for various consumption goods. The consumption bundles selected were food, housing, apparel and upkeep, health and recreation, transportation, and other goods and services. Finally, one month holding rates of returns for U.S. government debt obligations ranging in maturity from one month to twenty years were used.

Results of Gouldey's tests using individual stocks indicate the following: 1) the intercept of the SMH is significantly positive, 2) the estimated premium for market risk is less than predicted by the model, and 3) the risk premiums due to unpredictable commodity price inflation and changing interest rates are as predicted by the model. As Gouldey points out, however, the results are suspect due to non-stationarity of many of the parameters. In addition, the results are not conclusive because of the presence of some specification error and also because the residuals may not be independent and identically distributed due to the presence of an industry factor. Therefore, in order to examine the effects of this possible dependence in the residuals, Gouldey repeats the tests using industry portfolios. When portfolio data are used the risk parameters tend to be more stable than those of individual stocks and

much of the risk peculiar to individual stocks is diversified away in portfolios. Results of these tests indicate that the implications of the Long CAPM cannot be rejected using industry portfolios. However, the premium earned for market risk is less than predicted by the model. This may be due to other types of risk associated with omitted commodities or bills that investors consider relevant which would cause the estimated market risk premium to be biased downward.

Although the Long model did not fare well in his tests, Gouldey states that the evidence does not necessarily refute the model, but instead may cast doubt upon the abilities of investors to correctly predict future inflation and interest rates. Again, however, the results of the tests using portfolio industry data indicate that average rates of returns on capital assets do contain risk premiums for systematic market risk and risk due to stochastic consumption and investment opportunities. Those results imply that in principle investors can use stocks and bills to create a hedge against commodity price inflation and rising interest rates.

Even though Long suggests that these "quasi-futures" contracts exist in principle, there needs to be further development and testing of these hedge portfolios. Gouldey attempted to obtain estimates of the costs of forming "quasi-futures" for food, housing, and transportation price indices, but he failed to provide any information concerning the composition of the contracts nor did he test the success of the portfolios as hedging instruments using subsequently observed data. This dissertation intensively examines the construction and composition of "quasi-futures" contracts for ten consumption goods over the period 1965-1976. The "quasi-futures" contracts are subsequently tested for their adequacy as hedging instruments against commodity price inflation.

CHAPTER III
METHODOLOGY FOR CREATING "QUASI-FUTURES" CONTRACTS

This chapter presents a general methodology for the construction of "quasi-futures" contracts for various consumption goods. In order that a stock-bill portfolio can be properly called a "quasi-futures" contract for a particular good k, it must have the following two properties: (1) it should have the same expected "with-dividend" value at time 1 as the good, $\tilde{\Pi}_{k1}$, and (2) its covariance with $(\tilde{V}_{M1}, \tilde{\Pi}_1, \tilde{B}_1)$ should be equal to the covariance of $\tilde{\Pi}_{k1}$ with $(\tilde{V}_{M1}, \tilde{\Pi}_1, \tilde{B}_1)$.

The first step will be to select the stocks and long-term bills in such a way that the covariance property is attained. Let C be a $[(N+T-1)X(K+T)]$ matrix of the covariances appearing in the equilibrium pricing equations (II-1) and (II-2). The elements of C are given in Figure III-1. Let Σ be the $[(K+T)X(K+T)]$ variance-covariance matrix of $(\tilde{V}_{M1}, \tilde{\Pi}_1, \tilde{B}_1)$ whose elements are presented in Figure III-2. Finally, let Ω be a $[(N+T-1)X(K+T)]$ matrix whose elements are given in Figure III-3. Note that the first N rows of Ω contain the coefficients taken from the multiple regression of equation (II-4) for each of the N stocks.

The matrices are C, Σ, and Ω are related such that

$$C = \Omega\Sigma. \qquad \text{(III-1)}$$

For example, this implies that the first element of C is

$$
C \equiv \begin{bmatrix} \underset{(N\mathrm{x}1)}{\underline{\sigma}_M} & \underset{(N\mathrm{x}K)}{\Phi} & \underset{N\mathrm{x}(T-1)}{\Gamma} \\ \\ \underset{(T-1)\mathrm{x}1}{\underline{S}_M} & \underset{(T-1)\mathrm{x}K}{\Psi} & \underset{(T-1)\mathrm{x}(T-1)}{\Sigma_B} \end{bmatrix}_{(N+T-1)\mathrm{x}(K+T)}
$$

$$
\left[\begin{array}{c|c|c}
\begin{bmatrix} COV(V_{11},V_{M1}) \\ \vdots \\ (\underline{\sigma}_M : N\mathrm{x}1) \\ \vdots \\ COV(V_{N1},V_{M1}) \end{bmatrix} &
\begin{bmatrix} COV(V_{11},\pi_{11}) & \cdots & COV(V_{11},\pi_{K1}) \\ \vdots & \Phi : COV(V_{j1},\pi_{k1}) & \vdots \\ \vdots & N\mathrm{x}K & \vdots \\ COV(V_{N1},\pi_{11}) & \cdots & COV(V_{N1},\pi_{K1}) \end{bmatrix} &
\begin{bmatrix} COV(V_{11},B_{12}) & \cdots & COV(V_{11},B_{1T}) \\ \vdots & \Gamma : COV(V_{j1},B_{1m}) & \vdots \\ \vdots & N\mathrm{x}(T-1) & \vdots \\ COV(V_{N1},B_{12}) & \cdots & COV(V_{N1},B_{1T}) \end{bmatrix} \\
\begin{bmatrix} COV(B_{12},V_{M1}) \\ COV(B_{13},V_{M1}) \\ \vdots \\ (\underline{S}_M : (T-1)\mathrm{x}1) \\ \vdots \\ COV(B_{1T},V_{M1}) \end{bmatrix} &
\begin{bmatrix} COV(B_{12},\pi_{11}) & \cdots & COV(B_{12},\pi_{K1}) \\ COV(B_{13},\pi_{11}) & \cdots & \vdots \\ \vdots & \Psi : COV(B_{1m},\pi_{k1}) & \vdots \\ \vdots & (T-1)\mathrm{x}K & \vdots \\ COV(B_{12},\pi_{11}) & \cdots & COV(B_{1T},\pi_{K1}) \end{bmatrix} &
\begin{bmatrix} COV(B_{12},B_{12}) & \cdots & COV(B_{12},B_{1T}) \\ COV(B_{13},B_{12}) & \cdots & \vdots \\ & \Sigma_B : COV(B_{1m},B_{1m}) & \vdots \\ & (T-1)\mathrm{x}(T-1) & \vdots \\ COV(B_{1T},B_{12}) & \cdots & COV(B_{1T},B_{1T}) \end{bmatrix}
\end{array}\right]
$$

Figure III-1
Elements of the Covariance Matrix C

$$
\Sigma \equiv \begin{bmatrix}
\underline{S_M'1} & \underline{1'\phi} & \underline{S_M} \\
(1\times 1) & (1\times K) & (1\times(T-1)) \\[6pt]
\phi'\underline{1} & \Sigma_\pi & \Psi' \\
(K\times 1) & (K\times K) & K\times(T-1) \\[6pt]
\underline{S_M} & \Psi & \Sigma_B \\
(T-1)\times 1 & (T-1)\times K & (T-1)\times(T-1)
\end{bmatrix}
$$

$$
\left[\begin{array}{c|c|c}
\begin{bmatrix}
COV(V_{M1},V_{M1}) \\
=VAR(V_M) \\
\underline{S_M'1} \\
1\times 1
\end{bmatrix}
&
\begin{bmatrix}
COV(V_{M1},\pi_{11}) & \cdots & COV(V_{M1},\pi_{K1}) \\
 & \underline{1}'\phi & \\
 & (1\times K) &
\end{bmatrix}
&
\begin{bmatrix}
COV(B_{12},V_{M1}) & \cdots & COV(B_{1T},V_{M1}) \\
 & \underline{S_M'} & \\
 & (1\times(T-1)) &
\end{bmatrix}
\\ \hline
\begin{bmatrix}
COV(V_{M1},\pi_{11}) \\
COV(V_{M1},\pi_{21}) \\
\vdots \\
\phi'\underline{1} \\
K\times 1 \\
\vdots \\
COV(V_{M1},\pi_{K1})
\end{bmatrix}
&
\begin{bmatrix}
COV(\pi_{11},\pi_{11}) & COV(\pi_{11},\pi_{12}) & COV(\pi_{11},\pi_{k1}) \\
COV(\pi_{21},\pi_{11}) & \cdots & \vdots \\
\vdots & \Sigma_\pi & \vdots \\
\vdots & K\times K & \vdots \\
COV(\pi_{K1},\pi_{11}) & \cdots & COV(\pi_{K1},\pi_{K1})
\end{bmatrix}
&
\begin{bmatrix}
COV(B_{12},\pi_{11}) & COV(B_{13},\pi_{11}) & COV(B_{1T},\pi_{11}) \\
COV(B_{12},\pi_{21}) & \cdots & \vdots \\
\vdots & \Psi':COV(B_{1m},\pi_{k1}) & \vdots \\
\vdots & K\times(T-1) & \vdots \\
COV(B_{12},\pi_{K1}) & & COV(B_{1T},\pi_{K1})
\end{bmatrix}
\\ \hline
\begin{bmatrix}
COV(B_{12},V_{M1}) \\
COV(B_{13},V_{M1}) \\
\vdots \\
\underline{S_M} \\
(T-1)\times 1 \\
\vdots \\
COV(B_{1T},V_{M1})
\end{bmatrix}
&
\begin{bmatrix}
COV(B_{12},\pi_{11}) & \cdots & COV(B_{12},\pi_{K1}) \\
COV(B_{13},\pi_{11}) & \cdots & \vdots \\
\vdots & \Psi:\ COV(B_{1m},\pi_{k1}) & \vdots \\
\vdots & (T-1)\times K & \vdots \\
COV(B_{1T},\pi_{11}) & \cdots & COV(B_{1T},\pi_{K1})
\end{bmatrix}
&
\begin{bmatrix}
COV(B_{12},B_{12}) & & COV(B_{12},B_{1T}) \\
COV(B_{13},B_{12}) & & \vdots \\
\vdots & \Sigma_B:\ COV(B_{1m},B_{1m}) & \vdots \\
\vdots & (T-1)\times(T-1) & \vdots \\
COV(B_{1T},B_{12}) & \cdots & COV(B_{1T},B_{1T})
\end{bmatrix}
\end{array}\right]
$$

Figure III-2
Elements of the Covariance Matrix Σ

$$\Omega = \begin{bmatrix} \underset{(N\times 1)}{\varepsilon} & \underset{(N\times K)}{\Xi} & \underset{(N\times T-1)}{\Delta} \\ \underset{(T-1)\times 1}{\underline{0}} & \underset{(T-1)\times K}{\underline{0}} & \underset{(T-1)\times(T-1)}{I} \end{bmatrix}$$

$$\left[\begin{array}{ccc}
\begin{bmatrix} \varepsilon_{1M} \\ \varepsilon_{2M} \\ \vdots \\ \underset{(N\times 1)}{\varepsilon} \\ \vdots \\ \varepsilon_{NM} \end{bmatrix} &
\begin{bmatrix} \xi_{11} & \xi_{12} & \cdots & \xi_{1K} \\ \xi_{21} & \xi_{22} & \cdots & \xi_{2K} \\ \vdots & & \underset{(N\times K)}{\Xi} & \vdots \\ \xi_{N1} & \xi_{N2} & \cdots & \xi_{NK} \end{bmatrix} &
\begin{bmatrix} \delta_{12} & \delta_{13} & \cdots & \delta_{1T} \\ \delta_{22} & \delta_{23} & \cdots & \delta_{2T} \\ \vdots & & \underset{N\times(T-1)}{\Delta} & \vdots \\ \delta_{N2} & \delta_{N3} & \cdots & \delta_{NT} \end{bmatrix} \\
\begin{bmatrix} 0 \\ 0 \\ \vdots \\ \underset{(T-1)\times 1}{\underline{0}} \\ \vdots \\ 0 \end{bmatrix} &
\begin{bmatrix} 0 & 0 & \cdots & 0 \\ 0 & 0 & \cdots & 0 \\ \vdots & & \underset{(T-1)\times K}{\underline{0}} & \vdots \\ 0 & 0 & \cdots & 0 \end{bmatrix} &
\begin{bmatrix} 1 & 0 & 0 & \cdots & 0 \\ 0 & 1 & 0 & \cdots & 0 \\ 0 & 0 & 1 & \cdots & \vdots \\ \vdots & & \underset{(T-1)\times(T-1)}{I} & & \vdots \\ 0 & 0 & \cdots & & 1 \end{bmatrix}
\end{array}\right]$$

Figure III-3
Elements of the Augmented Coefficient Matrix Ω

$$\text{cov}(\tilde{V}_{11}, \tilde{V}_{M1}) = \beta_{1M}\text{var}(\tilde{V}_{M1}) + \sum_{k=1}^{K} \xi_{jk}\, \text{cov}(\tilde{V}_{M1}, \tilde{\Pi}_{k1})$$

$$+ \sum_{m=2}^{T} \delta_{jm}\text{cov}(\tilde{B}_{1m}, \tilde{V}_{M1}), \qquad \text{(III-2)}$$

which is equivalent to the covariance of equation (II-4) with $\tilde{V}_{M1}$. If the value of the stock market portfolio is unrelated to the price of goods or bills, then equation (III-2) will reduce to the more traditional expression as given in the single-period model for the covariance of the value of a stock with the market portfolio.[1]

---

[1]To demonstrate this point let

$\tilde{Y}_i = \alpha + \beta_1\tilde{X}_{1i} + \beta_2\tilde{X}_{2i} + \tilde{\varepsilon}_i.$

Our estimate for the vector of regression coefficients, $\hat{\beta}$, is

$$\hat{\beta} = \begin{bmatrix} X_1'X_1 & X_1'X_2 \\ X_2'X_1 & X_2'X_2 \end{bmatrix}^{-1} \begin{bmatrix} X_1'Y \\ X_2'Y \end{bmatrix},$$

or equivalently

$$\hat{\beta} = \begin{bmatrix} (X_1'M_2X_1)^{-1} & -(X_1'M_2X_1)^{-1}X_1'X_2(X_2'X_2)^{-1} \\ -(X_2'X_2)^{-1}X_2'X_1(X_1'M_2X_1)^{-1} & (X_2'X_2)^{-1}+(X_2'X_2)^{-1}A(X_2'X_2)^{-1} \end{bmatrix} \begin{bmatrix} X_1'Y \\ X_2'Y \end{bmatrix}$$

where

$A = X_2'X_1(X_1'M_2X_1)^{-1}X_1'X_2$; and

$M_2 = I - X_2(X_2'X_2)^{-1}X_2'$.

Now, if 1) $X_2'X_2=0$ (a trivial case), or 2) $X_1'X_2=0$, which would imply for this paper that the market portfolio (X_1) was orthogonal to the vector of good and bill prices (X_2), then our estimate of $\hat{\beta}_1$ would reduce to

$\hat{\beta}_1 = (X_1'X_1)^{-1}X_1'Y,$

the traditional measure of the risk of a security relative to the market portfolio.

It can be shown that the sum of the first N rows of C is equal to the first row of Σ by applying the following weighting scheme:

$$[1' \vdots 0'] \; C = [1' \vdots 0'] \; \Omega\Sigma$$

$$= [1 \; 0 \; 0 \; \ldots \; 0] \; \Sigma$$

$$= e_1{}'\Sigma , \qquad \text{(III-3)}$$

where 1' is a (1XN) vector of ones and 0' is a [1X(T-1)] vector of zeros. The above relationship implies the following:

$$\sum_{j=1}^{N} \beta_{jm} = 1, \qquad \text{(III-4)}$$

$$\sum_{j=1}^{N} \xi_{jk} = 0, \; k=1,\ldots,K, \qquad \text{(III-5)}$$

$$\sum_{j=1}^{N} \delta_{jm} = 0, \; m=2,\ldots,T. \qquad \text{(III-6)}$$

The validity of these equations can be demonstrated by summing equation (II-4) across all N securities and by recalling the definition of the value of the market portfolio, $\tilde{V}_{M1} \equiv \sum_{j=1}^{N} \tilde{V}_j$. These conditions, (III-4), (III-5), and (III-6), must hold during the empirical analysis in the next chapter in order to verify that the estimation procedure used is correct.

Let e'_{1+k} be a [1X(K+T)] vector whose (1+k)th element is 1, with all other elements equal to zero. The index k on e'_{1+k}, k=1,...,K, indicates that e'_{1+k} is being associated with the kth consumption good. For example, when k=0, vector $e_1{}'$ will refer to the "market" as was previously shown in (III-3). The next step is to find the k weighted combinations of the rows of Ω that will solve for the K row vectors,

e'_{1+k}, k=1,...,K. These weights can be found by solving the following system of equations for b'_k where b'_k is a $[1X(N+T-1)]$ vector:

$$b'_k \, \Omega = e'_{1+k}$$

$$b'_k \, \Omega \, \Omega^{-1} = e'_{1+k} \, \Omega^{-1}$$

$$b'_k = e'_{1+k} \, \Omega^{-1}, \; k=1,\ldots,K. \tag{III-7}$$

Each of the K row vectors, b'_k, can be interpreted as the unit quantities of the respective shares of common stock and long-term bills which correspond to a portfolio whose "with-dividend" value at time 1 has a covariance with the vector $(\tilde{V}_{M1}, \tilde{\Pi}_1, \tilde{B}_1)$ equal to the covariance of $\tilde{\Pi}_{k1}$ with $(\tilde{V}_{M1}, \tilde{\Pi}_1, \tilde{B}_1)$. The first N elements of b'_k refer to the unit quantities of shares of each stock and the last (T-1) elements of b'_k refer to the number of each of the long-term bills.

Now that the covariance property has been attained, the next step adjusts the expected value of each of the K stock-bill portfolios to equal $E(\tilde{\Pi}_{k1})$, the expected next period's value of the kth good. To set the expected value of the portfolio equal to $E(\tilde{\Pi}_{k1})$, a quantity of short-term bills (m=1) can be added to or subtracted from the portfolio without affecting its covariance with $(\tilde{V}_{M1}, \tilde{\Pi}_1, \tilde{B}_1)$. This is possible since the estimated regression coefficients from (II-4) that are used in solving for b'_k are independent of the price of a one-period bill. Therefore, the quantity of short-term bills, Y_{01k}, to add to the hedging portfolio is

$$Y_{01k} = B_{01} \, (\bar{\Pi}_{k1} - b'_k \, [\bar{P}_1 \quad \bar{B}_1]'), \; k=1,\ldots,K. \tag{III-8}$$

A negative value of Y_{01k} refers to a short position in one-period bills or a borrowing of funds at the short-term bill rate. Thus, a mean-adjusted

portfolio with a current value of F_{k0} can be created and is referred to as a "quasi-futures" contract for good k. A simple example to help clarify the methodology used in solving for the vector of portfolio weights, b_k', is presented in Appendix A for an economy containing the shares of two firms, one consumption good, and one long-term bill. It will be the convention during the empirical analysis in this paper to select N, the number of different stocks in the market portfolio, to be equal to one plus the number of consumption goods in the economy. That is, N=K+1. By doing so, there will exist only one "quasi-futures" contract per good each period. If N>K+1, then there will exist an infinite number of portfolios which could serve as "quasi-futures" contracts for a particular good.[2] However, of all such portfolios, only that portfolio which had the minimum variance about its next period's "with-dividend" expected value would be chosen as the "quasi-futures" contract for a particular good.

When constructing the hedging portfolios, it is interesting that the number of long-term bills in the economy assumed does not impose any empirical restrictions on the model as does the number of shares and consumption goods assumed. Also, it is shown in Appendix B that the number of shares of each firm held in the hedge portfolio does not depend directly on the regression coefficients, Δ, associated with the long-term bills. However, one can not conclude that bills are unimportant in calculating the share weights. Exclusion of bills in the multiple regression (II-4) would change the values of β and Ξ and thus would have an effect on the share weights.

[2]There will exist N-K linear independent solutions. Any linear combination of these solutions will also be a solution.

CHAPTER IV
THE FORMATION OF "QUASI-FUTURES" CONTRACTS

IV. A The Data

In order to select the proper quantities of stocks and long-term bills such that the covariance property of each "quasi-futures" contract is attained, the first step is to run the multiple regression of equation (II-4). The following time series data are required to perform the regression: (1) monthly prices for N=K+1 common stocks, (2) monthly prices for the value of the stock market portfolio, (3) monthly prices for K consumption goods, and (4) monthly prices for T-1 default-free long-term bills. The price data for the above assets were collected for the period January 1961 through March 1976. An explanation for the choice of this time period will be discussed later in this section.

Previous studies concerning commodity price inflation (Dusak (1973) and Gouldey (1977)) were somewhat limited in the number of different commodities used. If several goods are used in the analysis, and all the goods were to experience the same relative price changes or inflation rates, then one would expect the same hedging portfolio to be derived for each good except for the amount of short-term bills needed to adjust the expected value of each good. However, it appears that all goods do not experience the same relative price changes (see Fama and Schwert, 1977b) and thus the composition of each "quasi-futures" contract should be different for each good. Therefore, to test the success of "quasi-futures" contracts properly, the study should include a wide variety of consumption goods in the analysis.

The data concerning the consumption goods to be used presented two main problems. First, to be theoretically consistent with the Long model, all consumption goods should be included in the analysis. Including all these variables as independent variables in the regression (II-4) may present severe multi-collinearity problems. However, failure to include enough of these variables may result in misspecification of the model and missing-variable problems. The second problem concerns finding accurate and available commodity price data. For this purpose prices of seven various groupings containing closely related bundles of consumption goods used by the Bureau of Labor Statistics in compiling the Consumer Price Index (CPI) will be used to proxy for commodity prices. Price indices for these various groupings are readily available. In addition, due to problems to be discussed in some of the CPI components, price indices for three components of the Wholesale Price Index (WPI) will be used in order to have a larger cross-section of commodity data.[1] The various component indices selected to proxy for commodity prices are given in Table IV-1.

In addition to the reasons previously discussed for not using a single good for the analysis, one would not try to use the overall CPI as a proxy to relect the price changes for a particular good. These reasons are: (1) periodic revisions in the weighting of subcomponents, (2) insensitivity of the CPI to price changes during periods of low mean rates of inflation due to rounding or truncation to one digit after the decimal, (3) differences in the accuracy of measurement across components, and

---

[1]The following discussion concerning the CPI components is taken heavily from Fama and Schwert (1977b).

Table IV-1
Selected Consumption Bundles

	Component	Source	Symbol
1.	Food	CPI	FD
2.	Gas and Electricity	CPI	GE
3.	Apparel and Upkeep	CPI	AU
4.	Private Transportation	CPI	PT
5.	Reading and Recreation	CPI	RR
6.	Medical Care	CPI	MC
7.	Household Furnishings	CPI	HFO
8.	Metal and Metal Products	WPI	MMP
9.	Lumber and Wood Products	WPI	LWP
10.	Chemicals and Allied Products	WPI	CAP

(4) problems in defining the overall inflation rate if relative price changes induce substitution effects causing consumers to consume goods in different proportions at different times.

To retain the uniqueness of each price series, monthly price data for the selected consumption bundles were used. Fama and Schwert (1979) show that inflation rates of different goods can be broken into a part common to all goods and a part peculiar to each good itself. An example of a part peculiar to a particular good is one that may be due to seasonals. If production for a good is seasonal whereas demand occurs smoothly throughout the year, then seasons in the price of such a good may be observed to offset the cost of storing the output. However, as one goes from using monthly prices to prices of longer intervals, the variability of the price series of a component becomes more and more like

that of the CPI. Thus, the variability of price indices of various goods will become more and more similar as they are measured over longer intervals.

Final selection of the consumption bundles used in this study depends in many ways on the construction of the CPI. An overall breakdown of the major groupings composing the CPI is given in Table IV-2. Since the data to be used in the study are on a monthly basis, it will be desirable to have the selected CPI components sampled and priced monthly. In addition, it is desirable for each component chosen to be a fairly homogeneous grouping of goods. Approximately 50 percent of the items and locations used to construct the CPI are sampled every month. The rest of the component prices are collected on a quarterly basis, but on a rotating basis so that there is some monthly revision of prices. However, components which are sampled on quarterly intervals will reflect changes in prices which actually occurred during the preceding two months. Therefore, some components at times will be incorrectly calculated as being unchanged. These lags in the updating of prices may introduce autocorrelation in the price series.

The Food component which represents 22.4 percent of the CPI is priced monthly in all locations and is probably the cleanest series used in the study. The Gas and Electricity index which is a subcomponent of the Fuel and Utilities index represents about 3 percent of the CPI and is also priced monthly in all locations. However, since most of this component depends on utility rates determined by governmental agencies, the behavior of its price series may be different had the prices been determined in a free market as the other components. Despite this shortcoming, this series was retained in the study due to its importance as an everyday consumption item.

Table IV-2
A Breakdown of the Consumer Price Index[1]

Component	Percent of All Items, Dec. 1963		
Food*			22.43
Food at Home		17.89	
Cereals and Bakery Products	2.45		
Meats, Poultry, and Fish	5.63		
Dairy Products	2.80		
Fruits and Vegetables	3.02		
Other Food	3.99		
Food Away from Home		4.54	
Housing			33.23
Shelter		20.15	
Homeownership	14.27		
Rent	5.50		
Hotels and Motels	.38		
Fuel and Utilities		5.26	
Fuel Oil and Coal	.73		
Gas and Electricity*	2.71		
Telephone and Water	1.82		
Household Furnishings and Operation*		7.82	
Apparel and Upkeep*			10.63

Component	Percent of All Items, Dec. 1963		
Transportation			13.88
Private Transportation*		12.64	
Autos and Related Goods	3.28		
Automobile Services	3.62		
Public Transportation		1.24	
Health and Recreation			19.45
Medical Care*		5.70	
Drugs	1.14		
Professional Services	2.59		
Hospital Services	.36		
Health Insurance	1.61		
Personal Care		2.75	
Reading and Recreation*		5.94	
Other Goods and Services		5.06	
Tobacco Products	1.89		
Alcoholic Beverages	2.64		
Personal Expenses	.53		
Miscellaneous			.38

[1] Condensed from Appendix Table IX of U.S. Department of Labor, Bureau of Labor Statistics, Bulletin No. 1517, "The Consumer Price Index: History and Techniques."

*Refers to those components included in the study.

It seemed desirable to include the Homeownership series in the list of components chosen for the study. However, there are many problems with this series due to its subcomponents such as Mortgage Interest Expenses and Real Estate Taxes. The expense incurred in financing a good is not a factor of its price. In addition, government agencies, and not a free market, determine real estate taxes. A third subcomponent of the Homeownership series is the Home Purchase Price Index. This index is a three month moving average of newly insured FHA housing prices and thus suffers from the problem of built-in lags. Finally, another shelter related index, Rent, was also not included in this study, since this index is based on contract rents for a fixed sample of apartments, where the rent for a given apartment is sampled every six months with all unsampled months assumed to be unchanged in price. In addition, since most rental contracts are negotiated on an annual basis, this component does not truly reflect the actual price changes in the rental market. Household Furnishings (about 7.8% of the CPI) is a fairly homogeneous grouping and was selected for the study since it did not suffer from the shortcomings previously discussed.

Apparel and Upkeep which comprises about 10.6% of the CPI is a fairly clean and homogenous grouping and was selected, as was Private Transportation. Private transportation (about 12.6% of the CPI) reflects the cost of new and used cars, and the price of gasoline, motor oil, and auto parts.

The Health and Recreation index is a very heterogeneous mix and contains many built-in lags. Two of its subcomponents, Medical Care (5.7%) and Reading and Recreation (5.9%) are much cleaner indices and were thus selected.

Due to the lack of correctly measured CPI components, three price indices from the Wholesale Price Index (WPI) were selected to enlarge the sample of goods used. The three selected series are Metal and Metal Products, Lumber and Wood Products, and Chemicals and Allied Products. Despite the fact that these series are not truly consumer prices, they are homogeneous groupings and appear to accurately reflect price movements in the particular goods.

In summary, monthly price data for the period January 1961 through March 1976 were gathered for K=10 consumption bundles taken from the Consumer Price Index and the Wholesale Price Index to be used as proxies for various commodity prices. One reason for selecting January 1961 as a starting point for the study was due to an important change in the way the CPI was constructed beginning in 1964. At this time the coverage of the index was extended and the weights assigned to the different items are updated in line with the 1960-1961 Consumer Expenditure Survey. However, it was still possible to gather revised data for the selected price series back to January 1961 according to the 1964 updating scheme.

One final point related to the commodity price indices concerns the Nixon wage and price controls. Beginning in August 1971, the Nixon administration instituted price controls which were not lifted until 1973-1974. Since the controls led to queues and shortages, reported inflation rates probably understated true changes in purchasing costs to consumers. After the controls were lifted, the reported inflation rates probably overstated the true inflation rates. Tests will be performed in Section V. B. to see if the controls had any significant effect on the success of the "quasi-futures" contracts as hedging devices.

Based on the decision to use K=10 commodities in the study, monthly price data on N=11 (K+1) common stocks are required. This empirical restriction that the number of stocks to be used equals one plus the number of goods was discussed in Section II. A. Recalling that if N>K+1, then there will exist an infinite number of hedging portfolios which have all the properties of a "quasi-futures" contract for a particular good. However, of all such portfolios, only that portfolio which had the minimum variance about its next period's "with-dividend" expected value would be chosen as the "quasi-futures" contract for a particular good. Despite not being the optimal approach for creating a "quasi-futures" contract, the problem of searching for a minimum variance portfolio will be avoided by setting N=K+1, whereby, only one solution is produced.

As Gouldey (1977) points out, the risk parameters using portfolio data tend to be more stable than those of individual stocks and much of the risk peculiar to individual stocks is diversified away in portfolios. Instead of using individual securities in the analysis, industry portfolios are formed.[2] Various industry classifications as provided by the Compustat files will be used to create the industry portfolios. The eleven industry groupings to be used are listed in Table IV-3. The industries selected are widely varied and an attempt was made to have all the industries correspond to each of the consumption bundles selected. For example, the "Motor Vehicles and Auto Trucks" portfolio corresponds to the Private Transportation component. The purpose of this is to see if the best hedging strategy against inflation in a particular good consists of buying the stock of a firm in a related industry.

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[2] It would be desirable to use individual stocks. However, due to the instability of the regression estimates when individual stocks are used, industry portfolios are created. It is realized that industry portfolios may not be fully diversified.

Table IV-3
Industry Portfolios

Name	Compustat Industry Code	Symbol
1. Chemicals-Major	2801	CH
Chemicals-Minor	2802	
2. Textile Apparel MGF	2300	TA
3. Forest Products	2400	FP
4. Home Furnishings	2510	HF
5. Drugs-Ethical	2835	D
Drugs-Proprietary	2836	
Drugs-Medical Supply	2837	
6. Steel-Major	3310	ST
Steel-Minor	3311	
7. Motor Vehicles	3711	MV
Auto Trucks	3713	
8. Elec Utilities-Normalized	4912	EU
Elec Utilities-Flow Thru	4911	
9. Retail-Food Chains	5411	F
10. Leisure Time Products	3948	LT
Amusement & Recreation	7949	
Prof. Sports & Arenas	7941	
11. Natural Gas Companies	4924	NG

A price index was created to represent a set of prices for each of the industry portfolios. The following procedure was used to create each price index. First, each company within an industry grouping was checked to see if it had a monthly return (RET1) listed in the monthly returns file of the Center for Research in Security Prices (CRSP) at the University of Chicago. For all companies that had a monthly return listed in a given month, the returns were summed and an average monthly return was computed for that industry. This procedure was repeated for each month from January 1961 through March 1976. Firms were frequently observed to enter and leave the sample when computing each industry average monthly return. This method allowed the maximum number of companies to be used each month in the calculation of the industry average monthly return.

The regression in equation (II-4) requires time series data in the form of prices rather than returns. To convert the eleven industry average return series into price indices, the following steps were taken for each industry. First, each industry was assigned a base value of 100 at the beginning of the time period. Then, the value of an industry portfolio in any subsequent month t was calculated by multiplying the price in month t-1 by one plus the portfolio return in month t. By performing these calculations for all months, eleven industry price indices were created.

To be consistent with the theory underlying the model, the value of the stock market portfolio in month t is defined to equal the sum of the values of all stocks in month t. That is $V_{Mt} = \sum_{j=1}^{N} V_{jt}$, for all months t. Therefore, a stock market price index was created by summing each industry price index across industries.

Finally, Long's model requires data for default-free coupon-less bills of all maturities. The prices of U.S. Government Treasury bills were

used because of their availability and reliability. The interest rate on Treasury bills was pegged by the Fed before 1951, thus not allowing Treasury bill rates to adjust to anticipated variation in inflation rates. Price data were available on bills with maturities from one to three months until March 1959, at which time they became available for maturities up to six months. Price data on Treasury bills with maturities up to twelve months became available beginning in August 1964. Since the data on the consumption bundles were collected back to 1961, monthly prices for Treasury bills with maturities ranging from one to six months were collected for the period January 1961 through March 1976.[3] It will be the convention to treat the one month bill as a short-term bill. The Treasury bills ranging in maturity from two to six months are treated as long-term bills. Thus, there are five (T-1) long-term bills and one short-term bill used in the study.

IV. B Empirical Estimation of the Regression Coefficients

Following the collection of the various monthly price data, the next step is to estimate the regression coefficients in the following multiple regression whose parameters were discussed in Chapter II:

$$\tilde{V}_{j1} = \alpha_j + \beta_{jM}\tilde{V}_{M1} + \sum_{k=1}^{K} \xi_{jk}\,\tilde{\Pi}_{k1} + \sum_{m=2}^{T} \delta_{jm}\tilde{B}_{1m} + \tilde{\varepsilon}_j, \quad j=1,\dots, N. \quad \text{(II-4)}$$

The above equation requires using realizations of time 1 values of the variables. Since we do not have perfect foresight, the empirical counterpart to equation (II-4) will be the following:

[3]See Fama (1975) for a description of the Treasury bill data used in this study.

$$E(\tilde{V}_{j1}) = \hat{\alpha}_j + \hat{\beta}_{jM}E(\tilde{V}_{M1}) + \sum_{k=1}^{K} \hat{\xi}_{jk}\, E(\tilde{\Pi}_{k1}) + \sum_{m=2}^{T} \hat{\delta}_{jm}E(\tilde{B}_{1m}) + \tilde{\omega}_j,$$

$$j = 1,\ldots, N. \qquad \text{(II-4a)}$$

A naive expectations model is used to estimate each variable's expected value. Thus, the convention in this study will be: (1) to use today's stock and commodity prices as estimates of next period's value; (2) to use today's value of the stock market portfolio as an estimate of next period's value; and (3) to use today's forward prices as implied by the term structure as the best estimates of next period's bill prices. Conventions (1) and (2) assume that stock and commodity prices follow a martingale. Convention (3) assumes the expectations theory of the term structure. Today's forward price for each bill is then computed according to

$$E(\tilde{B}_{1m}) = B_{01}^{-1}\, B_{0m}, \quad m = 2,\ldots, T. \qquad \text{(IV-1)}$$

The number of observations used in each of the N=11 regressions performed over each time interval must exceed the number of coefficients (K+T+1 = 17) to be estimated. Five years of monthly observations will be used in each time series regression. The first set of N regressions will be performed on sixty months of observations extending from January 1961 through December 1965. The estimated coefficients from this time interval are then used to compute the one period or one month "quasi-futures" contracts for each of the goods according to the method discussed in Chapter III. Thus, the first sixty months of observations are used to create portfolios which serve as hedging devices against price inflation in the various selected goods during January 1966. Since the "quasi-futures" contracts are one-period contracts, new estimates of the regression coefficients must be computed each month in order to rebalance

the portfolios. This is done by repeating the regression after first dropping the observations from the oldest month and adding observations from the latest month. By using this updating scheme, observations from the most recent sixty months are used in each regression. For example, the portfolio weights of "quasi-futures" contracts for goods to be delivered in February 1966 are computed using the estimated regression coefficients from the period February 1961 through January 1966.

During the initial runs of the regressions, serial correlation in the residuals was found. In order to get unbiased estimates of the regression coefficients, the data were transformed into first differences. When the method of first differences is used, it is believed that rho, the first order serial coefficient of the regression disturbances, is close to unity. In order to justify the assumption that the value of rho is sufficiently close to unity, the Hildreth-Lu[4] maximum likelihood scanning procedure was used in regressions over various selected time intervals. This technique transforms the data by rho (e.g.: $X_t - \rho X_{t-1}$) and selects the value of rho which results in the lowest transformed error sum of squares. A sample of the estimated values of ρ for each of the eleven regressions over the interval January 1961 through December 1965 is presented in Table IV-4. Since ρ was sufficiently close to unity in the sample regressions, the method of first differences was used in all regressions.[5] In addition, the constant term was omitted from the regressions since there is no reason to include a trend variable.

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[4]See Hildreth and Lu (1960) for a description of this procedure.

[5]It is assumed that the residuals follow a first-order autoregressive process. Since the values of rho are sufficiently close to unity, a first-difference transformation of equation (II-4a) is actually used to estimate the regression coefficients.

Table IV-4
Estimation of Equation II-4a:
Hildreth-Lu Scanning Technique
January 1961 to December 1965

Dependent Variable	RHO	R^2
CH	.99	.9948
TA	.97	.9955
FP	.80	.9957
HF	.91	.9958
D	.97	.9893
ST	.82	.9779
MV	.99	.9918
EU	.95	.9927
F	1.00	.9703
LT	.70	.9620
NG	.81	.9824

In order to verify that the estimation procedure used to calculate the regression coefficients is correct, three conditions must hold over each time interval. Recalling from Chapter III, these conditions are as follows:

$$\sum_{j=1}^{N} \beta_{jm} = 1, \tag{III-4}$$

$$\sum_{j=1}^{N} \xi_{jk} = 0, \quad k = 1,\ldots,K, \tag{III-5}$$

$$\sum_{j=1}^{N} \delta_{jm} = 0, \quad m = 2,\ldots,T. \tag{III-6}$$

These equalities exist due to the definition of the value of the market index. The estimation procedure used proved to be quite accurate as demonstrated in Table IV-5, which presents a summary of the regression coefficients and their totals across industries from the eleven regressions over the sample interval March 1971 through February 1976.

IV. C Computation and Analysis of the Portfolio Weights

Following the estimation of each set of regression coefficients, the next step is to create the matrix Ω from which the monthly portfolio weights are computed. The matrix Ω is a $[(N+T-1) \times (K+T)]$ matrix whose elements were given in Figure III-3. For each time interval, the regression coefficients from each set of $N = 11$ regressions are placed in the first N rows of Ω. The last 5 (T-1) rows are completed as given in Figure III-3.

Recalling from Chapter III, let e'_{1+k} be a $[1\mathrm{x}(K+T)]$ vector whose (1+k)th element is 1 with all other elements equal to zero. The index k on e'_{1+k}, k=1,...,K, indicates that the vector e'_{1+k} is associated with the kth consumption good. The next step is to find the K = 10 weighted combinations of the rows of Ω that will solve for the K row vectors,

Table IV-5
Summary of Regression Coefficients
March 1971 through February 1976

Dependent Variable		Independent Variables						
	MKT	FD	GE	AU	PT	RR	MC	MMP
CH	.0669	- .5701	-2.3750	- .7372	1.9180	1.132	- .8702	- .0143
TA	.1071	- .8521	2.0410	.2575	1.4100	-3.926	1.2730	.7788
FP	.1988	2.7260	-3.4410	- .5543	- .3889	-1.900	5.0020	-1.4560
HF	.1161	- .5773	2.7910	-1.6700	- .5573	-7.307	-2.0060	1.6700
D	.0885	- .3146	-3.4510	3.5780	-2.1110	9.460	-6.8900	-3.5690
ST	.0645	- .2655	- .4873	-2.8510	-1.0870	-1.386	3.4470	1.2850
MV	.0556	.2306	- .1272	-1.4650	- .3603	-3.106	2.2070	.9150
EU	.0309	.1880	2.0550	.4193	.4532	5.152	-2.2470	-1.7060
F	.0560	- .0127	3.2380	- .3316	.0398	-2.291	-1.4820	.5178
LT	.1862	- .4138	-2.0370	4.7080	1.0590	3.006	1.3380	3.4300
NG	.0294	- .1388	1.7940	-1.3540	- .3752	1.166	.2279	-1.8520
Total	1.0000	- .0003	.0005	- .0003	.0003	.000	- .0003	- .0007

Table IV-5 (Continued)

LWP	CAP	HFO	B2	B3	B4	B5	B6
.7809	- .2589	5.8090	-17.000	22.110	-30.510	19.570	2.872
- .6554	-1.3200	- .6824	-24.050	-21.750	8.614	- 8.703	12.810
1.4560	3.3560	-6.2300	-10.860	22.820	-17.010	13.430	-17.600
- .8505	-1.0730	2.7040	8.951	-23.750	12.590	-14.440	3.898
.8151	1.9480	3.6340	27.060	24.200	10.840	2.884	6.659
.1725	.0006	1.9880	- 3.252	- 9.809	11.650	-18.210	14.840
- .0111	- .6112	- .1225	- 1.083	-13.780	6.869	- 7.158	1.016
- .0461	.2280	-2.2000	33.040	18.490	- 3.089	- 2.089	-10.890
- .4022	- .2441	-1.5190	2.304	- .644	- 1.085	-10.520	9.836
-1.4730	-2.0370	-4.8020	-66.740	-34.730	2.564	45.060	-21.140
.2137	.0120	1.4220	51.630	16.840	- 1.431	-19.820	- 2.304
- .0001	.0004	.0011	.000	- .003	.002	.004	- .003

e'_{1+k}, $k = 1, \ldots, 10$, from the following system of equations:

$$b'_k = e'_{1+k}\,\Omega^{-1}, \quad k = 1, \ldots, 10. \tag{III-7}$$

The vector b'_k then contains the unit quantities of the respective shares and long-term bills which correspond to a portfolio whose "with-dividend" value one month later has a covariance with the vector $(\tilde{V}_{M1}, \tilde{\Pi}_1, \tilde{B}_1)$ equal to the covariance of $\tilde{\Pi}_{k1}$ with $(\tilde{V}_{M1}, \tilde{\Pi}_1, \tilde{B}_1)$.

After attaining the correct covariance properties for each stock-bill portfolio, the expected value for next month is set equal to the next month's expected value of the good for which it is hedging. Adding a quantity of bills with one month to maturity to the portfolio accomplishes this. These short-term bills will not disturb the covariance of the portfolio with $(\tilde{V}_{M1}, \tilde{\Pi}_1, \tilde{B}_1)$. The quantity of one-month bills, Y_{01k}, to add to each portfolio is

$$Y_{01k} = B_{01}\,(\bar{\Pi}_{k1} - b'_k\,[\bar{P}_1\;\bar{B}_1]'), \quad k = 1, \ldots, 10.^{6} \tag{III-8}$$

A negative value of Y_{01k} refers to a short position in one month bills. After the quantity of one month bills is computed and added to each portfolio, a "quasi-futures" contract for each good is created which will have all the same desirable risk and return properties that concern investors as the goods themselves.

The "quasi-futures" contracts for each good are of one month in length. Their portfolio weights are calculated using the regression coefficients from the sixty most recent months of price observations.

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[6] The bars over the variables indicate time 1 expectations.

Therefore, every month starting from December 1965 and extending to February 1976, a new "quasi-futures" contract is constructed for each good for delivery one month later.[7]

The quantities of each of the shares and bills in the hedging portfolios for a particular good changed significantly from month to month, thus indicating a need for rebalancing. Table IV-6 presents a typical example of the fluctuations in the portfolio weights of the "quasi-futures" contracts for Food (FD) over the interval September 1969 through December 1969. As one might expect, the fluctuations are more apparent in the quantities of the various bills due to their high degree of correlation and substitution. The next chapter investigates the need for rebalancing further. Fama and Schwert (1977a) investigate the question of which assets provided effective hedges against inflation during the 1953-1971 period. By regressing the returns on various assets against both the anticipated and unanticipated components of the inflation rate, they conclude that U.S. Government bonds and bills were a complete hedge against expected inflation and that common stocks were negatively related to the expected component of the inflation rate, and probably also to the unexpected component. These results imply that bills serve as a better hedge against inflation than do common stocks during the 1953-1971 inflationary period. In addition, the "quasi-futures" contracts which serve as hedging devices should therefore consist mostly of bills. In fact, if returns on common stock were negatively related to the inflation rate, one would expect to observe a short position in common stocks each month.

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[7]Tables specifying the composition of each "quasi-futures" contract are available from the author upon request.

Table IV-6
Composition of "Quasi-Futures" Contracts for Food[1]
September 1969 through December 1969

Industries	9/69	10/69	11/69	12/69
	Months			
CH	- .217	- .160	.468	.525
TA	.014	.099	.044	.063
FP	- .269	- .310	.194	.012
HF	- .139	- .237	- .069	- .104
D	- .035	- .029	- .077	- .093
ST	.264	.777	.560	.627
MV	.244	- .121	- .940	- 1.053
EU	- .453	- .025	.094	.083
F	.718	.863	.202	.279
LT	.063	.071	.071	.086
NG	.546	- .417	- .701	- .752
Bills[2]				
2-Month	10.085	18.990	1.880	3.816
3-Month	-13.168	-30.131	-34.815	-40.306
4-Month	-40.013	-39.643	9.057	8.498
5-Month	1.261	-12.041	-14.245	-17.365
6-Month	11.987	29.916	17.706	20.889
1-Month[3]	29.960	33.716	22.342	26.443

[1]Numbers in table represent quantities of the respective shares of stock and bills.

[2]Bills are assumed to have a face value of $100.

[3]1-month bills are used only for adjusting the expected value of the portfolio.

Examining the percentage of each of the two types of assets in each monthly portfolio confirms these implications. Table IV-7 presents the fraction of the total dollar investment in common stock and in bills in each of the monthly "quasi-futures" contracts for the consumption bundle "Lumber and Wood Products" from December 1965 through February 1976. In all but 14 of the 123 months there was a larger dollar investment in bills. In fact, in more than half of the months there was a negative dollar investment or a short position in common stock. The results for all of the other consumption goods were similar to those for "Lumber and Wood Products." Thus it appears that bills do serve as a better hedge against commodity price inflation than do common stocks.

While bills were a better hedging mechanism than common stocks, no conclusion can be drawn as to which of the bill maturities is better. Each of the various bills were frequently observed to be held short from month to month. Finally, the results imply that purchasing the common stock of a firm that produces a good similar to the hedged commodity would not contribute to the hedging efficiency of the portfolio.

Table IV-7
Fraction of the Total Dollar Investment in Stocks and Bills in the "Quasi-Futures" Contracts for Lumber and Wood Products
December 1965 through February 1976

MONTH	1965 STOCKS	1965 BILLS	1966 STOCKS	1966 BILLS	1967 STOCKS	1967 BILLS	1968 STOCKS	1968 BILLS
JAN			-.042	1.042	.010	.990	21.072	-20.072
FEB			-.033	1.033	.045	.955	-.706	1.706
MAR			-.072	1.072	.540	.460	-.344	1.344
APR			-.040	1.040	.185	.815	-.515	1.515
MAY			.003	.997	-.625	1.625	.056	.944
JUN			.002	.998	-.595	1.595	-.930	1.930
JUL			-.004	1.004	-.474	1.474	-1.529	2.529
AUG			.075	.925	-.390	1.390	1.997	-.997
SEP			-.026	1.026	-.022	1.022	-.416	1.416
OCT			.013	.987	.231	.769	-.428	1.428
NOV			-.029	1.029	-1.285	2.285	-.414	1.414
DEC	-.004	1.004	-.013	1.013	.122	.878	-.304	1.304

Table IV-7 (Continued)

1969		1970		1971		1972	
STOCKS	BILLS	STOCKS	BILLS	STOCKS	BILLS	STOCKS	BILLS
-.768	1.768	-1.058	2.058	-.004	1.004	.314	.686
-.282	1.282	-.160	1.160	.161	.839	-.360	1.360
-.406	1.406	-.022	1.022	.319	.681	-.087	1.087
-.969	1.969	.005	.995	.314	.686	.038	.962
-.666	1.666	-.057	1.057	.359	.641	.032	.968
.148	.852	-.794	1.794	.268	.732	-.075	1.075
-.957	1.957	-.089	1.089	.271	.729	-.158	1.158
-.669	1.669	-.079	1.079	3.981	-2.981	-.865	1.865
-.281	1.281	-.097	1.097	2.155	-1.155	.496	.504
.645	.355	-.102	1.102	-.350	1.350	-.080	1.080
-.303	1.303	-.026	1.026	.870	.130	-.336	1.336
-.787	1.787	.008	.992	2.576	-1.576	.807	.193

Table IV-7 (Continued)

1973		1974		1975		1976	
STOCKS	BILLS	STOCKS	BILLS	STOCKS	BILLS	STOCKS	BILLS
.759	.241	-.239	1.239	.247	.753	-.050	1.050
-.129	1.129	2.285	-1.285	.468	.532	-.001	1.001
-.088	1.088	.242	.758	.404	.596		
-.146	1.146	.068	.932	.347	.653		
-.362	1.362	.026	.974	.347	.653		
-1.005	2.005	.104	.896	5.736	-4.736		
-.112	1.112	.326	.674	.370	.630		
-3.320	4.320	.899	.101	.381	.619		
1.724	-.724	-2.607	3.607	.175	.825		
-.237	1.237	4.064	-3.064	.039	.961		
-.378	1.378	.636	.364	.045	.955		
-.574	1.574	-.552	1.552	.197	.803		

CHAPTER V
TESTS OF THE PORTFOLIOS AS HEDGING INSTRUMENTS

Following their construction, the monthly "quasi-futures" contracts are tested to determine their success as hedging devices against commodity price inflation. As discussed previously, a "quasi-futures" contract for a particular good is constructed such that its expected value at the delivery date is equal to $\bar{\Pi}_{k1}$. In addition, the portfolio should also have a covariance with $(\tilde{V}_{M1}, \tilde{\Pi}_1, \tilde{B}_1)$ equal to the covariance of $\tilde{\Pi}_{k1}$ with $(\tilde{V}_{M1}, \tilde{\Pi}_1, \tilde{B}_1)$.

V. A Testing Price and Risk Characteristics

Two tests were designed to investigate whether each series of "quasi-futures" contracts were indeed true substitutes for owning the goods themselves. The first of these tests was to indicate if the subsequently observed value at delivery of each "quasi-futures" contract was identical to the observed value of the good at the same delivery date. This was accomplished by testing the significance of the estimated regression coefficients in the following time series regressions for each of the 10 consumption goods:

$$F_{kt} = \gamma_{k0} + \gamma_{k1} \Pi_{kt} + \varepsilon_{kt}, \; k = 1,\ldots,10, \qquad \text{(V-1)}$$

where F_{kt} is the actual observed value at delivery date t of a "quasi-futures" contract which was purchased one month earlier. The value of F_{kt} at delivery is computed according to

$$F_{kt} = b'_{k,t-1}\,[P_t\ B_t]' + 100\,Y_{t-1,tk}, \quad k = 1,\ldots,10. \qquad \text{(V-2)}$$

Let a perfect hedge be defined as one where the hedging portfolio and the good have identical observed values each period. This would require the intercept and slope coefficients in equation (V-1) to be exactly zero and one, respectively, with zero standard errors. In addition, the R^2 of each regression should equal one. Let an unbiased hedge be defined as one in which the estimated slope and intercept coefficients are not statistically different from zero and one, respectively. Thus, the hypothesis of whether the "quasi-futures" contracts do indeed have the same subsequently observed values as the goods is tested by examining the regression coefficients in equation (V-1) which should have the following expected values:

$$E(\tilde{\gamma}_{k0}) = 0, \qquad k = 1,\ldots,10,$$

and

$$E(\tilde{\gamma}_{k1}) = 1, \qquad k = 1,\ldots,10.$$

Estimates of the regression coefficients in equation.(V-1) from the period January 1966 to March 1976 are presented in Table V-1 for each of the consumption goods. The estimates of the intercepts coefficients, γ_{k0}, k=1,...,10, are all within one standard error from zero, with three exceptions, as indicated by the t statistics. The three exceptions are Gas and Electricity (t = 1.04), Apparel and Upkeep (t = -1.71), and Household Furnishings (t = -1.18). However, none of the t values are significant at a .05 significance level. Similarly, the estimates of the slope coefficients, γ_{k1}, k=1,...,10, are all within one standard error from one, with one exception (Apparel and Upkeep: t = 1.62). Again,

Table V-1
Portfolios as Substitutes for Consumption Goods:
Ordinary Least Squares

$$F_{kt} = \gamma_{k0} + \gamma_{k1}\Pi_{kt} + \varepsilon_{kt}$$

January 1966 - March 1976, T=123
(standard errors in parenthesis)

Good	$\hat{\gamma}_{k0}$	$t(\gamma_{k0}=0)^a$	$\hat{\gamma}_{k1}$	$t(\gamma_{k1}=1)^a$	Durbin-Watson	R^2	$F(\gamma_{k0}=0,\ \gamma_{k1}=1)^b$
FD	-19.03 (31.25)	-.61	1.15 (.24)	.63	2.26	.16	.19
GE	27.63 (26.33)	1.04	.83 (.21)	-.81	2.36	.11	1.29
AU	-53.39 (31.20)	-1.71	1.42 (.26)	1.62	2.00	.20	1.80
PT	.08 (26.86)	.00	1.01 (.23)	.04	2.16	.14	.04
RR	8.84 (14.68)	.60	.91 (.12)	-.75	2.32	.31	.89
MC	-13.96 (22.75)	-.61	1.07 (.18)	.39	2.15	.23	.99
MMP	21.58 (24.67)	.87	.87 (.19)	-.68	2.35	.15	.70
LWP	80.32 (88.75)	-.90	1.37 (.63)	.59	2.21	.04	1.18
CAP	-13.25 (14.57)	-.91	1.11 (.12)	.92	2.25	.41	.43
HFO	-16.59 (14.12)	-1.18	1.10 (.12)	.83	2.07	.43	1.53

[a]Critical $t_{.05,121} = 1.980$

[b]Critical $F_{.05;2,121} = 3.07$

none of the t values are significant at a .05 significance level. Finally, an F test can be conducted (see Theil, p. 133) which will test simultaneously the joint hypothesis that $\gamma_{k0} = 0$ and $\gamma_{k1} = 1$ for each good. This statistic is determined according to

$$F_{2,T-2}(\hat{\gamma}_{k0}, \hat{\gamma}_{k1}) = [\hat{\gamma}_{k0} - \gamma_{k0} \quad \hat{\gamma}_{k1} - \gamma_{k1}] \, (s^2C)^{-1} \, [\hat{\gamma}_{k0} - \gamma_{k0} \quad \hat{\gamma}_{k1} - \gamma_{k1}]'/2$$

where s^2C is the variance-covariance matrix of the regression coefficients. The values of F with 2 and 121 degrees of freedom for each regression are clearly insignificant at a .05 significance level as indicated in Table V-1.

The intercept and slope coefficients in equation (V-1) are re-estimated using seemingly unrelated estimation. This procedure will provide better estimates whenever the residual terms are contemporaneously correlated across regressions. Table V-2 presents these estimates of the intercept and slope coefficients along with the t statistics. None of the t values are significant at a .05 significance level. Table V-2 also presents an F statistic which differs from the previously described F statistic. This F statistic tests the general linear hypothesis that the intercept and slope coefficients across regressions are jointly equal to zero and one, respectively; see Zellner (1962). The F value is clearly insignificant at a .05 significance level.

The evidence supports the claim that each hedging portfolio and consumption good had statistically equivalent observed values each month. Although the results indicate that the portfolios were not perfect substitutes for the goods, the portfolios did serve as unbiased hedges against commodity price inflation in each good.

Table V-2
Portfolios as Substitutes for Consumption Goods:
Seemingly Unrelated Estimation

$$F_{kt} = \gamma_{k0} + \gamma_{k1}\,\Pi_{kt} + \varepsilon_{kt}$$

January 1966-March 1976, T = 123
(standard errors in parenthesis)

Good	$\hat{\gamma}_{k0}$	$t(\gamma_{k0}=0)^a$	$\hat{\gamma}_{k1}$	$t(\gamma_{k1}=1)^a$
FD	-19.60 (25.64)	-.76	1.15 (.20)	.75
GE	16.72 (17.74)	.94	.92 (.14)	-.57
AU	-46.36 (25.89)	-1.79	1.36 (.22)	1.64
PT	-10.28 (18.84)	-.55	1.10 (.16)	.63
RR	3.37 (11.24)	.30	.96 (.09)	-.44
MC	-11.50 (16.67)	-.69	1.05 (.13)	.38
MMP	11.24 (16.06)	.70	.96 (.12)	-.33
LWP	-69.71 (53.25)	-1.31	1.30 (.36)	.83
CAP	-9.74 (12.50)	-.78	1.08 (.10)	.80
HFO	-14.09 (11.46)	-1.23	1.09 (.09)	1.00

[a]Critical $t_{.05,121} = 1.980$

$F_{20,1210} = .69$; Critical $F_{.05} = 1.58$

The second step to indicate whether the "quasi-futures" contracts were true substitutes for the consumption goods is to test whether the relative risk properties of the stock-bill portfolios were similar to that of the goods. That is, the "quasi-futures" contract for good k should have a covariance with $(\tilde{V}_{M1}, \tilde{\Pi}_1, \tilde{B}_1)$ equal to the covariance of $\tilde{\Pi}_{k1}$ with $(\tilde{V}_{M1}, \tilde{\Pi}_1, \tilde{B}_1)$. This can be stated as

$$\mathrm{COV}\ \{\tilde{F}_{k1}, (\tilde{V}_{M1}, \tilde{\Pi}_1, \tilde{B}_1)\} = \mathrm{COV}\ \{\tilde{\Pi}_{k1}, (\tilde{V}_{M1}, \tilde{\Pi}_1, \tilde{B}_1)\},\ k = 1,\ldots,K. \tag{V-3}$$

Since the previous tests indicated that the portfolios had equivalent observed values at each delivery date, then any differences in the actual monthly values should be uncorrelated with $(\tilde{V}_{M1}, \tilde{\Pi}_1, \tilde{B}_1)$. If equation (V-3) is true then the following can be stated:

$$\mathrm{COV}\ \{(\tilde{F}_{kt} - \tilde{\Pi}_{kt}), (\tilde{V}_{M1}, \tilde{\Pi}_1, \tilde{B}_1)\} = 0, \quad k = 1,\ldots,K. \tag{V-4}$$

Letting D_{kt} be equal to $F_{kt} - \Pi_{kt}$, the difference in the observed monthly prices between the hedging portfolio and good k, the following set of 16 (K+T) regressions is run for each good k, k = 1,...,K.

$$D_{kt} = \alpha^1_{k0} + \alpha^1_{k1} V_{Mt} + \mu^1_{kt},$$

$$D_{kt} = \alpha^{j+1}_{k0} + \alpha^{j+1}_{k1} \Pi_{jt} + \mu^{j+1}_{kt}, \qquad j = 1,\ldots,K,$$

$$D_{kt} = \alpha^{K+i}_{k0} + \alpha^{K+i}_{k1} B_{t,t+i-1} + \mu^{K+i}_{kt}, \qquad i = 2,\ldots,T. \tag{V-5}$$

If the differences, D_{kt}, are truly uncorrelated with $(\tilde{V}_{M1}, \tilde{\Pi}_1, \tilde{B}_1)$, then all the intercept and slope coefficients in equation (V-5) should equal zero. Thus, the hypothesis to be tested is

$$H_0 : \alpha^1_{k0} = \alpha^2_{k0} = \ldots = \alpha^{K+T}_{k0} = 0$$

$$\alpha^1_{k1} = \alpha^2_{k1} = \ldots = \alpha^{K+T}_{k1} = 0, \quad k = 1, \ldots K.$$

An F test can be conducted (see Maddala, p. 323) which will test the above hypothesis. The F ratio is

$$F = \frac{(S_2 - S_1)/\,2(K+T)}{S_1/\left[\sum_{i=1}^{K+T} M_i - 2(K+T)\right]} .$$

where

1) M_i is the number of observations in the ith regression,

2) S_1 = the sum of the unrestricted residual sum of squares from each regression

$$= \sum_{i=1}^{K+T} RSS_i \text{ with df } = \sum_{i=1}^{K+T} M_i - 2(K+T),$$

and

3) S_2 = the restricted residual sum of squares from a pooled regression of the data

$$= (K+T) \sum_t D^2_{kt} \text{ with df } = \sum_{i=1}^{K+T} M_i .$$

Estimates of the regression coefficients in equation (V-5) from the period January 1966 to March 1976 are presented in Tables V-3 through V-12 for each of the 10 consumption goods. In addition, the t statistics for the coefficients are given along with the R^2 and the residual sum of squares from each regression. When trying to reject a null hypothesis, selecting a larger significance level produces a more stringent test. For all of the consumption goods, with the exception of Apparel and Upkeep, none of the estimated intercept or slope coefficients were greater than

Table V-3
Covariance Properties of Contracts for Food

$$F_{1t} - \Pi_{1t} = \alpha_{10} + \alpha_{11}\ (\text{Ind Var})_t + \mu_{1t}$$

January 1966 - March 1976, T = 123

Ind. Var.	$\hat{\alpha}_{10}$	$t(\alpha_{10}=0)$	$\hat{\alpha}_{11}$	$t(\alpha_{11}=0)$	R^2	Residual Sum of Squares
MKT	12.10	.37	-.01	-.38	.001	613715.5
FD	-19.03	-.61	.15	.61	.003	612553.1
GE	-15.30	-.47	.12	.47	.002	613356.6
AU	-17.10	-.33	.14	.33	.001	613909.9
PT	-22.75	-.51	.19	.51	.002	613139.1
RR	-17.20	-.34	.14	.34	.001	613878.3
MC	-12.62	-.36	.10	.36	.001	613803.4
MMP	-14.74	-.53	.11	.53	.002	613009.0
LWP	- 9.49	-.34	.07	.34	.001	613870.7
CAP	-21.39	-.79	.18	.81	.005	611166.4
HFO	-20.12	-.50	.17	.50	.002	613194.2
B1	5878.04	.87	-59.01	-.87	.006	610615.2
B2	1591.37	.51	-16.05	-.51	.002	613124.4
B3	779.82	.39	- 7.91	-.39	.001	613687.4
B4	924.96	.62	- 9.43	-.62	.003	612501.1
B5	795.53	.67	- 8.15	-.67	.004	612180.4

S_1 = 9807704.7   df = 1936

S_2 = 9831382.4   df = 1968

$F_{32,1936} = .15$

Critical $F_{.05} = 1.45$

Table V-4
Covariance Properties of Contracts for Gas and Electricity

$$F_{2t} - \Pi_{2t} = \alpha_{20} + \alpha_{21}(\text{Ind Var})_t + \mu_{2t}$$

January 1966 - March 1976, T = 123

Ind. Var.	$\hat{\alpha}_{20}$	$t(\alpha_{20}=0)$	$\hat{\alpha}_{21}$	$t(\alpha_{21}=0)$	R^2	Residual Sum of Squares
MKT	10.05	.38	-.01	-.10	.000	394886.6
FD	26.91	1.08	-.16	-.80	.005	392847.4
GE	27.63	1.05	-.17	-.79	.005	392914.3
AU	47.07	1.14	-.34	-.97	.008	391882.3
PT	34.82	.98	-.24	-.78	.005	393949.7
RR	42.69	1.05	-.30	-.88	.006	392406.2
MC	30.79	1.10	-.19	-.85	.006	392567.7
MMP	24.78	1.11	-.14	-.81	.005	392816.6
LWP	24.95	1.12	-.13	-.81	.005	392784.9
CAP	18.61	.86	-.10	-.54	.002	393982.2
HFO	32.54	1.01	-.21	-.79	.005	392887.3
B1	1182.28	.21	-11.25	-.21	.000	394782.8
B2	-631.33	-.25	6.44	.26	.001	394708.3
B3	-628.49	-.39	6.45	.40	.001	394413.2
B4	-426.53	-.36	4.42	.36	.001	394493.0
B5	-290.78	-.31	3.05	.31	.001	394603.4

$S_1 = 6295925.9$ df = 1936

$S_2 = 6424353.6$ df = 1968

$F_{32,1936} = 1.23$

Critical $F_{.05} = 1.45$

Table V-5
Covariance Properties of Contracts for Apparel and Upkeep

$$F_{3t} - \Pi_{3t} = \alpha_{30} + \alpha_{31}(\text{Ind Var})_t + \mu_{3t}$$

January 1966 - March 1976, T=123

Ind. Var.	$\hat{\alpha}_{30}$	$t(\alpha_{30}=0)$	$\hat{\alpha}_{31}$	$t(\alpha_{31}=0)$	R^2	Residual Sum of Squares
MKT	.16	.01	-.01	-.21	.000	227742.5
FD	-33.89	-1.80	.24	1.62	.021	222999.0
GE	-28.48	-1.43	.20	1.25	.013	224913.8
AU	-53.40	-1.71	.42	1.59	.021	223136.2
PT	-43.92	-1.63	.34	1.50	.018	223679.6
RR	-47.71	-1.56	.37	1.44	.017	223986.7
MC	-34.78	-1.64	.24	1.48	.018	223780.1
MMP	-28.23	-1.68	.19	1.48	.018	223772.9
LWP	-27.24	-1.62	.17	1.42	.016	224116.7
CAP	-25.32	-1.55	.18	1.34	.015	224477.2
HFO	-39.11	-1.60	.29	1.46	.017	223884.7
B1	8791.19	2.18	-88.30	-2.18	.038	219232.9
B2	3993.25	2.15	-40.32	-2.15	.037	219441.1
B3	2224.94	1.85	-22.59	-1.85	.028	221568.3
B4	1704.25	1.90	-17.40	-1.90	.029	221169.8
B5	1509.15	2.12	-15.50	-2.13	.036	219606.8

$S_1 = 3567508.3$ df = 1936

$S_2 = 3677036.8$ df = 1968

$F_{32,1936} = 1.86$

Critical $F_{.05} = 1.45$

Table V-6
Covariance Properties of Contracts for Private Transportation

$$F_{4t} - \Pi_{4t} = \alpha_{40} + \alpha_{41} \text{ (Ind Var)}_t + \mu_{4t}$$

January 1966 - March 1976, T=123

Ind. Var.	$\hat{\alpha}_{40}$	$t(\alpha_{40}=0)$	$\hat{\alpha}_{41}$	$t(\alpha_{41}=0)$	R^2	Residual Sum of Squares
MKT	12.72	.64	-.01	-.60	.003	222115.1
FD	-2.75	-.15	.03	.20	.000	222696.9
GE	- .75	-.04	.01	.09	.000	222758.7
AU	6.08	.20	-.04	-.16	.000	222724.6
PT	.08	.00	.01	.04	.000	222771.6
RR	4.32	.14	-.03	-.11	.000	222752.0
MC	2.54	.12	-.01	-.07	.000	222763.9
MMP	1.44	.09	-.01	-.03	.000	222772.6
LWP	-6.18	-.37	.05	.44	.002	222417.0
CAP	-1.99	-.12	.03	.19	.000	222707.2
HFO	1.11	.05	-.01	-.00	.000	222773.8
B1	5749.06	1.42	-57.71	-1.42	.017	219103.4
B2	2737.57	1.47	-27.60	-1.47	.018	218843.8
B3	1480.92	1.23	-15.00	-1.23	.012	220015.1
B4	1025.41	1.15	-10.44	-1.15	.011	220380.1
B5	800.91	1.12	-8.19	-1.12	.010	220477.0

$S_1 = 3548072.8$ df = 1936

$S_2 = 3566366.8$ df = 1968

$F_{32,1936} = .31$

Critical $F_{.05} = 1.45$

Table V-7
Covariance Properties of Contracts for Reading and Recreation

$$F_{5t} - \Pi_{5t} = \alpha_{50} + \alpha_{51}\,(\text{Ind Var})_t + \mu_{5t}$$

January 1966 - March 1976, T=123

Ind. Var.	$\hat{\alpha}_{50}$	$t(\alpha_{50}=0)$	$\hat{\alpha}_{51}$	$t(\alpha_{51}=0)$	R^2	Residual Sum of Squares
MKT	5.09	.53	-.01	-.73	.004	51656.0
FD	2.02	.22	-.03	-.42	.002	51804.2
GE	3.91	.41	-.05	-.60	.003	51725.1
AU	8.37	.56	-.09	-.68	.004	51684.1
PT	7.14	.55	-.08	-.69	.004	51675.2
RR	8.84	.60	-.09	-.73	.004	51655.3
MC	5.11	.50	-.05	-.69	.004	51679.7
MMP	3.53	.44	-.04	-.67	.004	51688.8
LWP	.32	.04	-.01	-.26	.001	51851.5
CAP	1.87	.24	-.03	-.47	.002	51784.6
HFO	5.82	.50	-.06	-.65	.004	51697.5
B1	2258.23	1.16	-22.69	-1.16	.010	51313.2
B2	1062.91	1.18	-10.74	-1.18	.012	51285.8
B3	808.43	1.40	-8.21	-1.40	.016	51053.8
B4	483.15	1.12	-4.94	-1.12	.010	51344.3
B5	377.80	1.10	-3.89	-1.10	.010	51363.5

S_1 = 825262.6   df = 1936

S_2 = 836043.2   df = 1968

$F_{32,1936} = .79$

Critical $F_{.05} = 1.45$

Table V-8
Covariance Properties of Contracts for Medical Care

$$F_{6t} - \Pi_{6t} = \alpha_{60} + \alpha_{61} \text{ (Ind Var)}_t + \mu_{6t}$$

January 1966 - March 1976, T=123

Ind. Var.	$\hat{\alpha}_{60}$	$t(\alpha_{60}=0)$	$\hat{\alpha}_{61}$	$t(\alpha_{61}=0)$	R^2	Residual Sum of Squares
MKT	9.18	.43	-.01	-.70	.004	258216.2
FD	-18.26	-.90	.10	.64	.003	258395.9
GE	-16.64	-.78	.09	.53	.002	258670.2
AU	-19.74	-.59	.12	.42	.002	258880.0
PT	-19.99	-.69	.12	.50	.002	258724.9
RR	-16.83	-.51	.10	.34	.001	259011.2
MC	-13.96	-.61	.07	.37	.001	258965.0
MMP	-15.64	-.87	.08	.57	.003	258565.6
LWP	-10.06	-.55	.03	.25	.001	259127.5
CAP	-19.28	-1.10	.12	.81	.005	257881.1
HFO	-18.95	-.72	.11	.52	.002	258693.0
B1	3686.31	.84	-37.06	-.84	.006	257751.4
B2	1873.08	.93	-18.95	-.93	.007	257413.4
B3	1323.96	1.02	-13.48	-1.02	.009	257038.9
B4	1049.99	1.09	-10.75	-1.09	.010	256723.9
B5	785.51	1.02	-8.10	-1.02	.009	257019.0

S_1 = 4131077.2 df = 1936

S_2 = 4209499.2 df = 1968

$F_{32,1936}$ = 1.15

Critical $F_{.05}$ = 1.45

Table V-9
Covariance Properties of Contracts for Metal and Metal Products

$$F_{7t} - \Pi_{7t} = \alpha_{70} + \alpha_{71}\,(\text{Ind Var})_t + \mu_{7t}$$

January 1966 - March 1976, T=123

Ind. Var.	$\hat{\alpha}_{70}$	$t(\alpha_{70}=0)$	$\hat{\alpha}_{71}$	$t(\alpha_{71}=0)$	R^2	Residual Sum of Squares
MKT	14.14	.48	-.01	-.30	.001	483556.9
FD	21.39	.77	-.13	-.58	.003	482556.2
GE	22.01	.75	-.14	-.57	.003	482595.6
AU	44.21	.97	-.33	-.85	.006	481038.2
PT	30.93	.78	-.22	-.65	.004	482235.4
RR	38.55	.85	-.28	-.74	.005	481722.8
MC	28.08	.90	-.18	-.74	.005	481744.7
MMP	21.58	.87	-.13	-.67	.004	482138.4
LWP	19.46	.79	-.10	-.58	.003	482584.1
CAP	14.38	.60	-.08	-.38	.001	483337.7
HFO	28.70	.80	-.19	-.66	.004	482197.3
B1	1855.61	.31	-18.57	-.31	.001	483530.1
B2	151.93	.06	-1.48	-.05	.000	483899.1
B3	14.31	.01	-.09	-.00	.000	483910.2
B4	-44.64	-.03	.51	.04	.000	483904.6
B5	-69.15	-.07	.77	.07	.000	483890.3

$S_1 = 7724841.6$ df = 1936

$S_2 = 7803676.8$ df = 1968

$F_{32,1936} = .55$

Critical $F_{.05} = 1.45$

Table V-10
Covariance Properties of Contracts for Lumber and Wood Products

$$F_{8t} - \Pi_{8t} = \alpha_{80} + \alpha_{81}\ (\text{Ind Var})_t + \mu_{8t}$$

January 1966 - March 1976, T=123

Ind. Var.	$\hat{\alpha}_{80}$	$t(\alpha_{80}=0)$	$\hat{\alpha}_{81}$	$t(\alpha_{81}=0)$	R^2	Residual Sum of Squares
MKT	-55.01	-.52	.01	.25	.001	6225374.
FD	-110.51	-1.11	.65	.84	.006	6192771.
GE	-106.49	-1.02	.64	.75	.005	6199474.
AU	-204.64	-1.25	1.48	1.08	.010	6169336.
PT	-144.83	-1.02	.99	.83	.006	6193731.
RR	-177.49	-1.10	1.26	.93	.007	6184320.
MC	-126.47	-1.14	.77	.89	.007	6188088.
MMP	-111.62	-1.26	.65	.96	.008	6181574.
LWP	-80.32	-.90	.37	.59	.003	6210587.
CAP	-92.44	-1.08	.54	.76	.005	6199034.
HFO	-142.03	-1.11	.94	.89	.007	6187793.
B1	3326.18	.15	-33.68	-.16	.000	6227339.
B2	3214.50	.32	-32.72	-.33	.001	6223068.
B3	2723.63	.43	-27.90	-.43	.002	6219045.
B4	2181.21	.46	-22.52	-.47	.002	6217445.
B5	2122.31	.56	-22.03	-.57	.003	6211973.

S_1 = 99230952 df = 1936

S_2 = 101330400.0 df = 1968

$F_{32,1936}$ = 1.28

Critical $F_{.05}$ = 1.45

Table V-11
Covariance Properties of Contracts for
Chemicals and Allied Products

$$F_{9t} - \Pi_{9t} = \alpha_{90} + \alpha_{91}\ (\text{Ind Var})_t + \mu_{9t}$$

January 1966 - March 1976, T=123

Ind. Var.	$\hat{\alpha}_{90}$	$t(\alpha_{90}=0)$	$\hat{\alpha}_{91}$	$t(\alpha_{91}=0)$	R^2	Residual Sum of Squares
MKT	-17.53	-.98	.01	.99	.008	178612.7
FD	-17.69	-1.05	.14	1.06	.009	178401.5
GE	-20.15	-1.14	.17	1.15	.011	178119.7
AU	-28.61	-1.03	.24	1.03	.009	178505.3
PT	-29.12	-1.21	.25	1.22	.012	177879.3
RR	-30.83	-1.13	.26	1.13	.011	178171.4
MC	-22.09	-1.17	.17	1.18	.011	178007.9
MMP	-12.73	-.85	.10	.86	.006	178971.8
LWP	-13.17	-.87	.09	.89	.006	178900.8
CAP	-13.25	-.91	.11	.92	.007	178800.2
HFO	-24.13	-1.11	.20	1.12	.010	178223.7
B1	-1146.73	-.31	11.51	.31	.001	179914.0
B2	-285.07	-.17	2.87	.17	.000	180017.5
B3	-141.67	-.13	1.43	.13	.000	180034.9
B4	-34.42	-.04	.35	.04	.000	180057.4
B5	-145.54	-.23	1.49	.23	.000	179984.2

$S_1 = 2862602.3$ df = 1936

$S_2 = 2881004.8$ df = 1968

$F_{32,1936} = .39$

Critical $F_{.05} = 1.45$

Table V-12
Covariance Properties of Contracts for Household Furnishings

$$F_{10t} - \Pi_{10t} = \alpha_{10,0} + \alpha_{10,1}\ (\text{Ind Var})_t + \mu_{10t}$$

January 1966 - March 1976, T=123

Ind. Var.	$\hat{\alpha}_{10,0}$	$t(\alpha_{10,0}=0)$	$\hat{\alpha}_{10,1}$	$t(\alpha_{10,1}=0)$	R^2	Residual Sum of Squares
MKT	-.05	-.00	-.01	-.30	.001	75796.2
FD	-15.31	-1.40	.09	1.11	.010	75092.4
GE	-13.28	-1.15	.08	.87	.006	75382.1
AU	-19.71	-1.09	.14	.90	.007	75344.1
PT	-18.36	-1.18	.13	.96	.008	75273.7
RR	-17.72	-1.00	.12	.81	.005	75442.9
MC	-14.20	-1.16	.08	.89	.007	75358.6
MMP	-11.92	-1.22	.07	.89	.007	75357.2
LWP	-11.62	-1.19	.06	.89	.006	75394.0
CAP	-13.15	-1.39	.08	1.06	.009	75158.8
HFO	-16.59	-1.18	.11	.94	.007	75299.5
B1	3573.56	1.52	-35.91	-1.52	.019	74430.6
B2	1734.89	1.60	-17.53	-1.61	.021	74266.1
B3	1086.31	1.56	-11.05	-1.56	.020	74356.0
B4	840.49	1.62	-8.60	-1.63	.021	74227.3
B5	646.29	1.56	-6.65	-1.57	.020	74336.4

S_1 = 1200515.9 df = 1936

S_2 = 1237188.8 df = 1968

$F_{32,1936}$ = 1.85

Critical $F_{.05}$ = 1.45

1.63 standard errors from zero. Thus, the hypothesis that each of the regression coefficients is individually equal to zero cannot be rejected as indicated by the critical t value of 1.658 at a .10 significance level with 121 degrees of freedom. Only three of the regressions pertaining to Apparel and Upkeep contained t values which were slightly greater than 2.

The hypothesis that all the intercept and slope coefficients are jointly equal to zero cannot be rejected for eight of the ten consumption goods as indicated by the F statistic at the bottom of each table. All the F values with the exception of Apparel and Upkeep (AU) and Household Furnishings (HFO) are insignificant at a .05 significance level. Finally, the values of R^2 in all the regressions are extremely small (the highest in any being .037) indicating almost no relationship between the difference in the value of the hedging portfolios and goods each period and the vector of prices $(\tilde{V}_{M1}, \tilde{\Pi}_1, \tilde{B}_1)$.

Despite two significant F values, the findings essentially indicate that each series of "quasi-futures" contracts had the same covariance with the vector of prices $(\tilde{V}_{M1}, \tilde{\Pi}_1, \tilde{B}_1)$ as did each good. In addition, the neglible values of R^2 in each regression indicate that any observed difference each month between the value of the portfolio and the good was non-systematically related to $(\tilde{V}_{M1}, \tilde{\Pi}_1, \tilde{B}_1)$.

V. B Effect of the Nixon Wage and Price Controls

On August 15, 1971, President Nixon announced his New Economic Policy intended to keep wage increases down to a 5.5 percent annual rate and also to limit price increases, and thus inflation, to a 2.5 percent annual rate. Economists thought that the 3 percent difference would result from a rise in productivity and thus curtail the rising inflation rate in addition to stimulating real growth. They also believed at the time that if the

inflation was of the cost-push variety, then controls could be a success since rising costs of production would be pushing prices higher. Under demand-pull inflation, economists thought controls would be a waste of time since the demand itself requires limiting. Although the freeze applied to prices and wages, prices of stocks and bonds were not included. Interest rates were not frozen, but bankers and other lenders were urged to hold the line. The impact of these wage and price controls on the ability of the "quasi-futures" contracts to serve as successful hedging instruments against commodity price inflation is examined next.

The controls which went into effect beginning on August 16, 1971, led to queues and shortages which caused reported inflation rates through the price indices to understate the true changes in purchasing costs to consumers. August 1971 will be used as the starting date for the controls. As the controls were lifted, the reported inflation rates probably overstated the true inflation rates. For instance, prices of many goods on the commodity exchanges frequently reached their permissible daily price increases within a few minutes of the opening of the exchanges. During the ending months of the controls, the Administration tried to stagger price increases by gradually lifting the controls from various industries at a time. December 1974 will be used as the ending date when the controls were lifted from almost all industries.

The tests described in Section V. A will be used again to examine the success of the portfolios as hedging devices during the period August 1971 through December 1974. Table V-13 presents the results of the first test which investigates whether the portfolios had the same observed value each month as each of the goods. The hypothesis is that the intercept coefficient in equation (V-1) is equal to zero and the

Table V-13
Portfolios as Substitutes for Consumption Goods:
Ordinary Least Squares

$$F_{kt} = \gamma_{k0} + \gamma_{k1}\Pi_{kt} + \varepsilon_{kt}$$

August 1971 - December 1974, T=41
(standard errors in parenthesis)

Good	$\hat{\gamma}_{k0}$	$t(\gamma_{k0}=0)^a$	$\hat{\gamma}_{k1}$	$t(\gamma_{k1}=1)^a$	Durbin-Watson	R^2	$F(\gamma_{k0}=0,\ \gamma_{k1}=1)^b$
FD	- 87.19 (102.23)	-.85	1.59 (.73)	.81	2.13	.11	.41
GE	24.45 (53.89)	.45	.84 (.42)	-.38	2.16	.09	.27
AU	-162.16 (122.20)	-1.33	2.24 (.96)	1.29	2.49	.12	.39
PT	-35.59 (122.77)	-.29	1.26 (.99)	.26	2.16	.04	.22
RR	-57.58 (111.95)	-.51	1.42 (.88)	.48	2.46	.06	1.05
MC	-153.63 (141.79)	-1.08	2.02 (1.02)	1.00	2.31	.09	1.17
MMP	6.17 (39.21)	.16	.95 (.28)	-.18	2.32	.23	.01
LWP	- .50 (217.49)	-.00	.86 (1.31)	-.11	2.38	.01	.30
CAP	23.08 (24.26)	.95	.81 (.20)	-.95	2.57	.29	.46
HFO	-53.15 (55.70)	-.95	1.37 (.44)	.84	2.56	.20	2.39

[a]Critical $t_{.10,39} = 1.684$

[b]Critical $F_{.05;2,39} = 3.23$

slope coefficient is equal to one. The estimates of γ_{k0}, k = 1,...,10, are all within one standard error from zero, with two exceptions, as indicated by the t statistics. None of the t values are significant for a critical value of t = 1.68 at a .10 significance level with 39 degrees of freedom. Similarly, the estimates of γ_{k1}, k = 1,...,10, are all within one standard error from one, with one exception. Again, none of the t values are significant at a .10 significance level. The values of F in each regression, which test the joint hypothesis that $\gamma_{k0} = 0$ and $\gamma_{k1} = 1$ for each good k, are all insignificant for a critical value of F = 3.23 at a .05 significance level with 2 and 39 degrees of freedom. Finally, the values of R^2 from each regression appear to be slightly lower than those from the January 1966 to March 1976 period.

The intercept and slope coefficients are re-estimated using seemingly unrelated estimation. Table V-14 presents these estimates of the intercept and slope coefficients along with the t statistics. None of the t values are significant at a .10 significance level. The F statistic which tests the general linear hypothesis that the intercept and slope coefficients across regressions are jointly equal to zero and one, respectively, is also presented in Table V-14. The F value is insignificant at a .01 significance level.

The evidence again supports the claim that each hedging portfolio and consumption good had statistically equivalent values each month. The wage and price controls appear to have had little or no effect on the success of the portfolios to serve as unbiased hedges for the consumption goods.

Table V-14
Portfolios as Substitutes for Consumption Goods:
Seemingly Unrelated Estimation

$$F_{kt} = \gamma_{k0} + \gamma_{k1} \Pi_{kt} + \varepsilon_{kt}$$

August 1971-December 1974, T = 41
(standard errors in parenthesis)

Good	$\hat{\gamma}_{k0}$	$t(\gamma_{k0}=0)^a$	$\hat{\gamma}_{k1}$	$t(\gamma_{k1}=1)^a$
FD	-59.06 (38.02)	-1.55	1.39 (.26)	1.50
GE	-29.75 (30.80)	-.97	1.26 (.24)	1.08
AU	-14.97 (65.79)	-.23	1.08 (.51)	.16
PT	3.80 (56.87)	.07	.95 (.45)	-.11
RR	-4.95 (47.84)	-.10	1.00 (.38)	.00
MC	14.97 (64.29)	.23	.81 (.46)	-.41
MMP	-8.34 (19.30)	-.43	1.06 (.13)	.46
LWP	-28.97 (48.93)	-.59	1.03 (.24)	.13
CAP	-6.29 (14.49)	-.43	1.06 (.12)	.50
HFO	-10.82 (22.48)	-.48	1.03 (.17)	.18

[a]Critical $t_{.10,39} = 1.684$

$F_{20,390} = 1.83$; Critical $F_{.01} = 1.92$

The second set of tests for the wage and price control period will check to see if the covariance properties of the stock-bill portfolios were similar to that of the goods. Tables V-15 through V-24 contain the results of regressing the monthly observed price difference between the hedging portfolios and each good against each of the variables in $(\tilde{V}_{M1}, \tilde{\Pi}_1, \tilde{B}_1)$. The hypothesis is that each of the intercept and slope coefficients in equation (V-5) should jointly equal zero. That is,

$$H_0: \alpha_{k0}^{1} = \alpha_{k0}^{2} = \dots = \alpha_{k0}^{K+T} = 0$$

$$\alpha_{k1}^{1} = \alpha_{k1}^{2} = \dots = \alpha_{k1}^{K+T} = 0, \qquad k = 1,\dots K.$$

In the tables for each good, none of the estimated intercept or slope coefficients are significantly different from zero as indicated by the critical t value of 2.02 at a .05 significance level with 39 degrees of freedom. The hypothesis that all the intercept and slope coefficients are jointly equal to zero cannot be rejected for nine of the ten consumption goods. With the exception of Household Furnishings (HFO), all the F values are insignificant at the .05 significance level. The findings essentially indicate that each series of "quasi-futures" contracts had the same covariance with the vector of prices $(\tilde{V}_{M1}, \tilde{\Pi}_1, \tilde{B}_1)$ as did each good. Any observed difference between the value of the portfolio and the good each month was non-systematically related to $(\tilde{V}_{M1}, \tilde{\Pi}_1, \tilde{B}_1)$.

Some care must be given in the interpretation of these results. The hedge portfolios appear to provide successful hedges during the price control period. However, if the price index numbers for the commodities do not reflect true transaction prices, the portfolios only provide hedges against changes in the government reported price levels and not hedges against actual price changes in the market place.

Table V-15
Covariance Properties of Contracts for Food
$F_{1t} - \Pi_{1t} = \alpha_{10} + \alpha_{11}(\text{Ind Var})_t + \mu_{1t}$
August 1971 - December 1974, T = 41

Ind. Var.	$\hat{\alpha}_{10}$	$t(\alpha_{10}=0)$	$\hat{\alpha}_{11}$	$t(\alpha_{11}=0)$	R^2	Residual Sum of Squares
MKT	28.07	.41	-.01	-.49	.006	250464.1
FD	-87.19	-.85	.59	.81	.017	247838.6
GE	-86.60	-.63	.63	.60	.009	249730.8
AU	-251.02	-1.05	1.93	1.30	.026	245385.5
PT	-124.66	-.71	.96	.68	.012	249030.9
RR	-263.71	-.90	2.04	.89	.020	247038.8
MC	-183.16	-.90	1.28	.88	.019	247134.0
MMP	-59.74	-.76	.39	.71	.013	248806.8
LWP	-70.89	-.74	.40	.70	.012	248897.8
CAP	-53.86	-.77	.41	.70	.013	248837.3
HFO	-139.22	-.83	1.05	.81	.016	247880.9
B1	3180.74	.30	-31.99	-.30	.002	251428.9
B2	1617.98	.35	-16.38	-.35	.003	251220.6
B3	1214.28	.41	-12.37	-.41	.004	250935.9
B4	1275.07	.57	-13.06	-.57	.008	249916.0
B5	1244.98	.70	-12.82	-.70	.013	248851.9

$S_1 = 3983398.8$ df = 624

$S_2 = 4048830.0$ df = 656

$F_{32,624} = .32$

Critical $F_{.05} = 1.47$

Table V-16
Covariance Properties of Contracts for Gas and Electricity

$$F_{2t} - \Pi_{2t} = \alpha_{20} + \alpha_{21}(\text{Ind Var})_t + \mu_{2t}$$

August 1971 - December 1974, T = 41

Ind. Var.	$\hat{\alpha}_{20}$	$t(\alpha_{20}=0)$	$\hat{\alpha}_{21}$	$t(\alpha_{21}=0)$	R^2	Residual Sum of Squares
MKT	-15.18	-.57	.01	.70	.012	38052.7
FD	17.76	.44	-.10	-.36	.003	38396.3
GE	24.45	.45	-.16	-.40	.004	38372.9
AU	37.10	.39	-.27	-.36	.003	38401.4
PT	23.70	.34	-.16	-.30	.002	38439.8
RR	35.52	.31	-.25	-.28	.002	38449.6
MC	30.18	.38	-.19	-.34	.003	38415.4
MMP	11.35	.37	-.06	-.27	.002	38456.2
LWP	11.42	.31	-.05	-.22	.001	38478.7
CAP	9.44	.34	-.05	-.23	.001	38475.5
HFO	21.32	.32	-.14	-.28	.002	38451.9
B1	-1274.99	-.31	12.84	.31	.002	38434.7
B2	-733.12	-.40	7.43	.40	.004	38366.2
B3	-491.35	-.42	5.02	.42	.005	38351.6
B4	-392.49	-.45	4.04	.45	.005	38327.7
B5	-150.71	-.22	1.58	.22	.001	38479.2

$S_1 = 614349.8$ df = 624

$S_2 = 623216.6$ df = 656

$F_{32,624} = .28$

Critical $F_{.05} = 1.47$

Table V-17
Covariance Properties of Contracts for Apparel and Upkeep

$$F_{3t} - \Pi_{3t} = \alpha_{30} + \alpha_{31}\ (\text{Ind Var})_t + \mu_{3t}$$

August 1971 - December 1974, T = 41

Ind. Var.	$\hat{\alpha}_{30}$	$t(\alpha_{30}=0)$	$\hat{\alpha}_{31}$	$t(\alpha_{31}=0)$	R^2	Residual Sum of Squares
MKT	42.28	1.22	-.01	1.37	.046	63210.2
FD	-76.39	-1.48	.52	1.41	.048	63039.3
GE	-79.81	-1.14	.59	1.09	.029	64299.5
AU	-162.16	-1.33	1.24	1.29	.041	63522.2
PT	-111.96	-1.26	.87	1.21	.036	63840.1
RR	-199.35	-1.35	1.54	1.32	.043	63425.3
MC	-137.93	-1.34	.96	1.30	.041	63507.0
MMP	-50.12	-1.26	.33	1.17	.034	64003.2
LWP	-76.75	-1.61	.44	1.53	.057	62487.3
CAP	-41.03	-1.14	.31	1.04	.027	64454.5
HFO	-102.80	-1.21	.77	1.16	.034	64026.1
B1	9228.92	1.75	-92.72	-1.75	.073	61436.8
B2	3753.98	1.62	-37.93	-1.62	.063	62059.7
B3	1881.52	1.25	-19.13	-1.25	.039	63698.7
B4	1538.71	1.36	-15.74	-1.37	.046	63217.9
B5	1425.97	1.60	-14.67	-1.61	.062	62124.5

S_1 = 1012352.3 df = 624

S_2 = 1072020.0 df = 656

$F_{32,624} = 1.15$

Critical $F_{.05} = 1.47$

Table V-18
Covariance Properties of Contracts for Private Transportation

$$F_{4t} - \Pi_{4t} = \alpha_{40} + \alpha_{41}(\text{Ind Var})_t + \mu_{4t}$$

August 1971 - December 1974, T = 41

Ind. Var.	$\hat{\alpha}_{40}$	$t(\alpha_{40}=0)$	$\hat{\alpha}_{41}$	$t(\alpha_{41}=0)$	R^2	Residual Sum of Squares
MKT	25.20	.53	-.01	-.60	.009	120573
FD	-50.93	-.72	.34	.68	.012	120269
GE	-35.19	-.37	.25	.34	.003	121333
AU	-65.44	-.39	.49	.37	.004	121260
PT	-35.59	-.29	.26	.27	.002	121466
RR	-90.73	-.44	.69	.43	.005	121116
MC	-53.56	-.38	.36	.36	.003	121294
MMP	-12.42	-.23	.07	.17	.001	121592
LWP	-97.07	-1.50	.57	1.46	.052	115348
CAP	-4.30	-.09	.01	.03	.000	121685
HFO	-24.68	-.21	.17	.19	.001	121579
B1	11964.60	1.67	-120.18	-1.67	.066	113604
B2	5422.77	1.74	-54.76	-1.74	.072	112958
B3	3324.03	1.65	-33.76	-1.65	.065	113752
B4	2174.45	1.42	-22.21	-1.43	.050	115653
B5	1781.66	1.47	-18.31	-1.47	.053	115267

$S_1 = 1898749$ df = 624

$S_2 = 1952864$ df = 656

$F_{32,624} = .56$

Critical $F_{.05} = 1.47$

Table V-19
Covariance Properties of Contracts for Reading and Recreation

$$F_{5t} - \Pi_{5t} = \alpha_{50} + \alpha_{51} \text{ (Ind Var)}_t + \mu_{5t}$$

August 1971 - December 1974, T = 41

Ind. Var.	$\hat{\alpha}_{50}$	$t(\alpha_{50}=0)$	$\hat{\alpha}_{51}$	$t(\alpha_{51}=0)$	R^2	Residual Sum of Squares
MKT	3.12	.12	-.01	-.31	.002	36312.5
FD	-30.62	-.79	.19	.67	.001	35988.2
GE	-23.42	-.45	.14	.36	.003	36281.4
AU	-43.01	-.47	.30	.41	.004	36240.3
PT	-16.58	-.25	.09	.18	.001	36271.1
RR	-57.58	-.51	.42	.47	.006	36193.2
MC	-37.05	-.47	.23	.41	.004	36240.3
MMP	-9.11	-.30	.03	.15	.001	36380.0
LWP	-51.12	-1.43	.28	1.31	.042	34862.4
CAP	-5.40	-.20	.01	.02	.000	36399.5
HFO	-18.16	-.28	.10	.21	.001	36359.1
B1	4172.33	1.04	-41.95	-1.04	.027	35415.3
B2	2000.95	1.15	-20.24	-1.15	.032	35207.1
B3	1436.35	1.29	-14.62	-1.29	.041	34911.2
B4	943.25	1.12	-9.67	-1.12	.031	35256.1
B5	723.24	1.08	-7.47	-1.09	.029	35331.6

$S_1 = 573749.3$ df = 624

$S_2 = 597444.3$ df = 656

$F_{32,624} = .81$

Critical $F_{.05} = 1.47$

Table V-20
Covariance Properties of Contracts for Medical Care

$$F_{6t} - \Pi_{6t} = \alpha_{60} + \alpha_{61}(\text{Ind Var})_t + \mu_{6t}$$

August 1971 - December 1974, T = 41

Ind. Var.	$\hat{\alpha}_{60}$	$t(\alpha_{60}=0)$	$\hat{\alpha}_{61}$	$t(\alpha_{61}=0)$	R^2	Residual Sum of Squares
MKT	25.63	.54	-.01	-.80	.016	120950.6
FD	-82.87	-1.17	.51	1.00	.025	119849.0
GE	-94.37	-.99	.64	.86	.019	120629.1
AU	-202.41	-1.21	1.49	1.14	.032	118980.5
PT	-106.13	-.87	.76	.77	.015	121105.4
RR	-210.45	-1.03	1.57	.97	.024	120023.2
MC	-143.62	-1.08	1.02	1.00	.025	119867.9
MMP	-54.94	-1.01	.31	.80	.016	120978.8
LWP	-79.72	-1.21	.41	1.03	.027	119665.0
CAP	-46.43	-.94	.29	.71	.013	121374.6
HFO	-111.95	-.96	.78	.86	.019	120658.9
B1	3968.58	.53	-39.97	-.53	.007	122048.3
B2	2411.99	.75	-24.47	-.75	.014	121200.2
B3	1781.24	.86	-18.20	-.86	.019	120637.2
B4	1476.00	.95	-15.18	-.96	.023	120124.3
B5	1054.09	.85	-10.94	-.86	.019	120651.3

S_1 = 1928744.3 df = 624

S_2 = 2062723 df = 656

$F_{32,624}$ = 1.35

Critical $F_{.05}$ = 1.47

Table V-21
Covariance Properties of Contracts for Metal and Metal Products

$$F_{7t} - \Pi_{7t} = \alpha_{70} + \alpha_{71}\,(\text{Ind Var})_t + \mu_{7t}$$

August 1971 - December 1974, T = 41

Ind. Var.	$\hat{\alpha}_{70}$	$t(\alpha_{70}=0)$	$\hat{\alpha}_{71}$	$t(\alpha_{71}=0)$	R^2	Residual Sum of Squares
MKT	-7.85	-.23	.01	.23	.001	62407.3
FD	-10.00	-.19	.07	.19	.001	62429.6
GE	1.24	.02	-.01	-.02	.000	62487.7
AU	5.11	.04	-.04	-.04	.000	62485.3
PT	11.24	.13	-.09	-.13	.000	62461.0
RR	-15.81	-.11	.12	.11	.000	62470.4
MC	-1.84	-.02	.01	.02	.000	62488.0
MMP	6.17	.16	-.05	-.17	.001	62444.5
LWP	-37.60	-.79	.23	.80	.016	61488.8
CAP	7.39	.21	-.06	-.22	.012	62411.1
HFO	8.87	.11	-.07	-.11	.000	62469.4
B1	2773.82	.52	-27.86	-.52	.007	62054.1
B2	1406.69	.61	-14.20	-.61	.009	61901.4
B3	1014.10	.68	-10.29	-.68	.012	61750.8
B4	600.30	.54	-6.13	-.54	.007	62029.4
B5	513.11	.58	-5.27	-.58	.009	61957.2

S_1 = 995736.0 df = 624

S_2 = 999851.1 df = 656

$F_{32,624}$ = .08

Critical $F_{.05}$ = 1.47

Table V-22
Covariance Properties of Contracts for Lumber and Wood Products

$$F_{8t} - \Pi_{8t} = \alpha_{80} + \alpha_{81}\,(\text{Ind Var})_t + \mu_{8t}$$

August 1971 - December 1974, T = 41

Ind. Var.	$\hat{\alpha}_{80}$	$t(\alpha_{80}=0)$	$\hat{\alpha}_{81}$	$t(\alpha_{81}=0)$	R^2	Residual Sum of Squares
MKT	-4.08	-.03	-.03	-.13	.000	1298446
FD	-93.97	-.40	.50	.30	.002	1295921
GE	-165.19	-.53	1.10	.45	.005	1292129
AU	-367.43	-.63	2.70	.63	.010	1286048
PT	-224.60	-.56	1.62	.50	.007	1290585
RR	-325.75	-.49	2.39	.45	.005	1292205
MC	-267.07	-.57	1.75	.52	.007	1289881
MMP	-134.36	-.76	.79	.63	.010	1285864
LWP	-.50	-.00	.14	-.10	.000	1298596
CAP	-120.35	-.75	.82	.61	.010	1286537
HFO	-241.80	-.63	1.71	.57	.008	1288075
B1	-13172.8	-.54	132.04	.54	.008	1289221
B2	-5385.3	-.51	54.11	.51	.007	1290454
B3	-3631.57	-.53	36.61	.53	.007	1289646
B4	1634.4	-.32	16.43	.32	.003	1295676
B5	-1065.81	-.26	10.69	.26	.002	1296789

$S_1 = 20666073$ df = 624

$S_2 = 21148416$ df = 656

$F_{32,624} = .46$

Critical $F_{.05} = 1.47$

Table V-23
Covariance Properties of Contracts for Chemicals and Allied Products

$$F_{9t} - \Pi_{9t} = \alpha_{90} + \alpha_{91}\ (\text{Ind Var})_t + \mu_{9t}$$

August 1971 - December 1974, T = 41

Ind. Var.	$\hat{\alpha}_{90}$	$t(\alpha_{90}=0)$	$\hat{\alpha}_{91}$	$t(\alpha_{91}=0)$	R^2	Residual Sum of Squares
MKT	-19.66	-.83	.01	.89	.020	29573.7
FD	41.24	1.18	-.29	-1.15	.033	29180.9
GE	50.84	1.08	-.39	-1.06	.028	29330.7
AU	93.02	1.12	-.72	-1.11	.031	29252.1
PT	62.89	1.04	-.50	-1.03	.026	29383.0
RR	122.67	1.22	-.96	-1.21	.036	29079.7
MC	84.96	1.22	-.60	-1.20	.036	29096.8
MMP	29.77	1.11	-.20	-1.08	.029	29297.1
LWP	34.94	1.07	-.21	-1.04	.027	29355.2
CAP	23.08	.95	-.19	-.92	.021	29533.4
HFO	61.12	1.06	-.47	-1.05	.027	29351.2
B1	-3072.94	-.84	30.87	.84	.018	29643.2
B2	-1023.48	-.64	10.34	.64	.010	29865.2
B3	-335.23	-.32	3.41	.32	.003	30095.4
B4	-439.32	-.57	4.49	.57	.008	29929.6
B5	-444.68	-.72	4.57	.72	.013	29776.0

$S_1 = 471743.2$ df = 624

$S_2 = 483588.1$ df = 656

$F_{32,624} = .49$

Critical $F_{.05} = 1.47$

Table V-24
Covariance Properties of Contracts for
Household Furnishings

$$F_{10,t} - \Pi_{10,t} = \alpha_{10,0} + \alpha_{10,1}(\text{Ind Var})_t + \mu_{10,t}$$

August 1971 - December 1974, T = 41

Ind. Var.	$\hat{\alpha}_{10,0}$	$t(\alpha_{10,0}=0)$	$\hat{\alpha}_{10,1}$	$t(\alpha_{10,1}=0)$	R^2	Residual Sum of Squares
MKT	12.58	.55	.01	-.85	.018	27542.6
FD	-46.20	-1.37	.29	1.19	.035	27074.4
GE	-48.58	-1.07	.33	.93	.022	27443.5
AU	-94.25	-1.18	.69	1.10	.030	27206.9
PT	-52.36	-.89	.37	.79	.016	27612.1
RR	-106.30	-1.09	.79	1.03	.026	27310.2
MC	-75.67	-1.12	.50	1.02	.026	27314.7
MMP	-26.99	-1.04	.15	.80	.016	27599.0
LWP	-52.27	-1.68	.28	1.49	.054	26546.1
CAP	-22.07	-.94	.13	.68	.012	27725.5
HFO	-53.15	-.95	.37	.84	.018	27550.4
B1	3749.76	1.07	-37.72	-1.07	.028	27253.6
B2	1932.97	1.27	-19.57	-1.27	.040	26934.6
B3	1338.18	1.37	-13.64	-1.37	.046	26753.8
B4	985.59	1.34	-10.12	-1.35	.045	26797.4
B5	757.59	1.30	-7.84	-1.31	.042	26873.2

S_1 = 435538 df = 624

S_2 = 476083.4 df = 656

$F_{32,624} = 1.82$

Critical $F_{.05} = 1.47$

V. C Quarterly and Semiannual Rebalancing

From the conclusions reached in the previous hypothesis testing, it appears that one can successfully create an unbiased hedge against commodity price inflation using shares of common stock and Treasury bills of various maturities. These hedging portfolios or "quasi-futures" contracts had the same observed price and risk characteristics each month as did the goods. Theoretically, since the portfolios serve as one month hedges, new portfolios must be created each month in order to maintain the proper hedge. One may speculate that some minor rebalancing each month is necessary due to the realization of events. An example of the fluctuations in the portfolio weights for a typical series of contracts was presented in Table IV-5. The portfolio weights appear to be very sensitive to small changes in the data used in the multiple regressions. Multicollinearity, due to the bills, was probably introduced into the regressions, since price movements of bills with close maturities are strongly interrelated. Whenever a high degree of multicollinearity is encountered, the estimates of the regression coefficients will be highly imprecise and non-stationary over successive time intervals. This could possibly explain the large observed changes in the portfolio weights each period.

Adjusting the portfolio weights each month by the indicated amounts would involve substantial transaction costs, but it might be possible to reduce these costs and see how important the shifts in the portfolios weights are, by reducing the frequency of the rebalancing. That is, the success of the "quasi-futures" contracts as hedging instruments will be examined when the stock-bill portfolios are rebalanced: 1) every 3 months

or at quarterly intervals; and 2) every 6 months or at semiannual intervals. The procedure will be to keep the same position in each stock as determined at the beginning of the interval until the interval elapses, at which time the portfolio is rebalanced based on the sixty previous months of price data. As for the bills, the proceeds from each bill when it matures will be reinvested in the bill of the longest maturity. That is, at maturity the funds from the one month bill will be placed in a six month bill.

A comparison of the observed values of the hedging portfolios and the goods will first be made for the quarterly rebalancing case and then for the semiannual case. Table V-25 contains the results of the time series regressions of equation (V-1) for the period January 1966 through March 1976. The hypothesis that $\gamma_{k0} = 0$ for each good is rejected for two of the ten goods (Apparel and Upkeep: t = -4.03, and Household Furnishings: t = -2.26) at a critical t value of 1.658 for a .10 significance level with 121 degrees of freedom. The hypothesis that $\gamma_{k1} = 1$ for each good is also rejected for the same two goods. The values of F, which test the joint hypothesis that $\gamma_{k0} = 0$ and $\gamma_{k1} = 1$ for each good are presented in the last column. The hypothesis is rejected for the two goods, Apparel and Upkeep and Household Furnishings, at a critical F value of 3.07 for a .05 significance level with 2 and 121 degrees of freedom. Finally, the values of the R^2 of each regression are lower than when the portfolios were rebalanced monthly.

Before concluding that the portfolios still serve as an unbiased hedge for eight of the ten goods, the values of the Durbin-Watson statistics should be carefully examined. The critical Durbin-Watson value at a .05 significance level is 1.65 for 123 observations. Thus, each of the

Table V-25
Portfolios as Substitutes for Consumption Goods:
Quarterly Rebalancing

$$F_{kt} = \gamma_{k0} + \gamma_{k1}\Pi_{kt} + \varepsilon_{kt}$$

January 1966 - March 1976, T=123
(standard errors in parenthesis)

Good	$\hat{\gamma}_{k0}$	$t(\gamma_{k0}=0)^a$	$\hat{\gamma}_{k1}$	$t(\gamma_{k1}=1)^a$	Durbin-Watson	R^2	$F(\gamma_{k0}=0, \gamma_{k1}=1)^b$
FD	-18.58 (49.89)	- .37	1.24 (.39)	.62	1.01	.08	.85
GE	19.91 (40.26)	.49	.93 (.33)	-.20	1.00	.06	1.40
AU	-235.43 (58.48)	-4.03	2.89 (.49)	3.85	.96	.22	8.91
PT	- 16.22 (46.07)	- .35	1.19 (.39)	.47	.83	.07	.55
RR	- 21.79 (28.09)	- .78	1.12 (.24)	.51	.92	.16	2.42
MC	- 15.06 (30.74)	- .49	1.04 (.24)	.17	1.21	.14	1.51
MMP	44.92 (43.37)	1.04	.72 (.33)	-.84	.67	.04	.84
LWP	- 37.55 (101.42)	- .37	.89 (.72)	-.15	.84	.01	2.53
CAP	- 24.06 (24.51)	- .98	1.22 (.20)	1.06	1.15	.23	.59
HFO	- 56.55 (25.06)	-2.26	1.41 (.21)	2.00	.98	.28	3.52

[a] Critical $t_{.10,121} = 1.658$

[b] Critical $F_{.05;2,121} = 3.07$

regressions exhibit positive autocorrelation of the residuals. This fact alone is sufficient to conclude that the "quasi-futures" contracts did not provide an unbiased hedge in the months they were not rebalanced. When the portfolio is not rebalanced in any given month, there is much information contained in the previous residual that is not being used by the investor to adjust the expected value of the portfolio to that of the good.

The results for the semiannual case are presented in Table V-26. The results indicate a further deterioration of the success of the portfolios as hedging instruments. The hypothesis that the estimated intercept and slope coefficients are equal to zero and one, respectively, can be rejected for nine of the ten goods. In addition, the values of the Durbin-Watson statistics are somewhat lower. It appears that rebalancing at quarterly or semiannual intervals in order to avoid frequent brokerage commissions is a poor strategy for an investor who wishes to maintain an unbiased hedge. The realization of events each month appears to significantly influence the portfolio weights.

Futures markets allow investors to maintain a perfect hedge against commodity price inflation. This dissertation demonstrates how an investor can create an unbiased hedge, on a monthly basis, using the markets for common stocks and Treasury bills. Thus, the amount contributed by futures markets to the overall completeness of financial markets is questionable. In addition, trading in futures contracts does not exist for many commodities. However, the techniques explored in this dissertation allow investors to hedge against price inflation for any commodity. In order to maintain an unbiased hedge, frequent rebalancing of the portfolios is required. Thus, futures markets may be economically valuable since they allow investors to more efficiently hedge in terms of transaction costs.

Table V-26
Portfolios as Substitutes for Consumption Goods:
Semiannual Rebalancing

$$F_{kt} = \gamma_{k0} + \gamma_{k1}\Pi_{kt} + \varepsilon_{kt}$$

January 1966 - March 1976, T=123
(standard errors in parenthesis)

Good	$\hat{\gamma}_{k0}$	$t(\gamma_{k0}=0)$[a]	$\hat{\gamma}_{k1}$	$t(\gamma_{k1}=1)$[a]	Durbin-Watson	R^2	$F(\gamma_{k0}=0,\gamma_{k1}=1)$[b]
FD	-165.51 (74.17)	-2.23	2.63 (.58)	2.81	.85	.15	7.52
GE	33.81 (58.81)	.57	1.07 (.48)	.15	.55	.04	7.48
AU	-443.56 (74.38)	-5.96	4.77 (.62)	6.08	.69	.33	18.33
PT	- 92.23 (75.02)	-1.23	2.03 (.63)	1.63	.52	.08	4.81
RR	- 15.63 (28.79)	- .54	1.21 (.24)	.88	.50	.17	3.15
MC	- 35.17 (53.25)	- .66	1.32 (.41)	.78	.56	.08	.47
MMP	50.16 (66.03)	.76	.90 (.50)	- .20	.47	.03	3.17
LWP	92.87 (171.18)	.54	-.44 (1.21)	-1.19	.51	.01	4.50
CAP	-162.73 (39.16)	-4.16	2.44 (.33)	4.36	1.30	.32	9.80
HFO	-132.94 (33.56)	-3.96	2.17 (.28)	4.18	.87	.34	9.65

[a]Critical $t_{.10,121} = 1.658$

[b]Critical $F_{.05;2,121} = 3.07$

CHAPTER VI
SUMMARY AND CONCLUSION

VI. A A Summary of This Research

This dissertation has investigated whether portfolios composed of default-free bills of various maturities and shares of common stocks exist that will allow an investor to hedge against commodity price inflation, without actually entering the futures markets. These hedging portfolios, or "quasi-futures" contracts, for a particular commodity were constructed such that they would have the same price and risk properties as the commodity itself, such as the expected price at delivery and covariances with the stock market, all other commodities, and long-term bills. The results indicate that an investor can create a stock-bill portfolio which will serve as an unbiased hedge against commodity price inflation. Thus, if markets are perfect and securities are traded costlessly, futures markets may be unnecessary since they do not provide an investor with a service that cannot already be duplicated in the existing markets for common stocks and Treasury bills.

The techniques explored in this dissertation allow investors to hedge against price inflation in any commodity. Previously, investors could only obtain perfect hedges against price inflation in commodities for which they could purchase a futures contract. Thus, hedging by means of "quasi-futures" contracts will be especially appealing to investors who wish to hedge against price inflation in commodities for which organized futures trading does not exist.

Examination of the portfolio weights needed to maintain each month's hedge indicated a need for frequent rebalancing. This extensive rebalancing, which would be costly to an investor, indicated that futures markets do provide a necessary service once the assumption of perfect markets is relaxed. In addition to the value of their information content (see Black (1976)), organized commodity exchanges are economically valuable since they provide a less expensive means for hedging against commodity price inflation versus the use of "quasi-futures" contracts.

Explanations by various authors for the necessity of futures markets were presented in Section I. B. Included among these were (1) the insurance viewpoint of Keynes; (2) the gambling casino viewpoint of Hardy; (3) the information content derived from the pattern of futures prices discussed in Black; and (4) Stoll's rationale that farmers or privately held firms are reluctant or unable to trade ownership claims on certain assets or production techniques with which they are endowed. The literature, however, contains little empirical analysis of the above viewpoints. Dusak examined the returns to holders of futures contracts and found evidence which was directly contrary to Keynes's viewpoint and only partially supportive of Hardy's.

Long (1974), in his development of a multi-period capital asset pricing model, uses an economy that not only includes a stock market, as do the traditional single-period models, but also includes a market for default-free bills of different maturities and many consumption goods whose future prices are uncertain. Long relates the price of an asset to not only the systematic market risk, but also to the risk due to changing consumption opportunities (inflation risk) and changing investment opportunities (interest rate risk). Implementation of Long's pricing equation

(see equation (II-3)) requires the development of a specific stock-bill portfolio. This stock-bill portfolio, which has a current ex-dividend price of F_{k0}, is constructed so as to have a time 1 "with-dividend" value equal to the expected time 1 price of a particular good, $\tilde{\Pi}_{k1}$, and also to have the same covariances with the elements of $(\tilde{V}_{M1}, \tilde{\Pi}_1, \tilde{B}_1)$ as does $\tilde{\Pi}_{k1}$. This hedging portfolio is referred to as "quasi-futures" contract for good k.

Chapter III presented a detailed methodology of how to create a "quasi-futures" contract for a particular commodity. The steps to be taken included: (1) performing time series regressions using historic information on stock, commodity, and Treasury bill price data; (2) inverting a matrix containing the above regression coefficients, and interpreting the kth row as being the unit quantities of a stock-bill portfolio that has the same covariances with the vector of prices $(\tilde{V}_{M1}, \tilde{\Pi}_1, \tilde{B}_1)$ as does the kth good; and (3) setting the expected value of each hedging portfolio equal to the expected value of the kth good by adding a quantity of short-term bills.

Section IV. A discussed the various assets used in the study for which data were collected. The commodity data consisted of seven consumption bundles used in constructing the Consumer Price Index and three components from the Wholesale Price Index. The selected commodity indices are listed in Table IV-1. Eleven various industry portfolios were created since the risk parameters using portfolio data are more stable than those of individual stocks. Table IV-3 lists the industries used which were selected from the Compustat classifications. Finally,

Treasury bills, with maturities from one month up to six months, were used to represent prices of default-free coupon-less debt obligations.

Estimation of the regression coefficients needed for the calculation of the portfolio weights was discussed in Section IV. B. Each time series regression consisted of observations on stock, good, and bill price data taken from the sixty most recent months. Problems with serial correlation in the residuals were encountered in the regressions. This problem was alleviated by using the method of first differences. The use of this technique was defended by showing that the values of the first order serial coefficients of the regressions were close to unity.

The "quasi-futures" contracts for each good were constructed to serve as one month hedges. In order to properly maintain each hedge, new portfolio weights were computed each month using the regression coefficients that were estimated from the sixty most recent months of price observations. Thus, every month from December 1965 through February 1976, new "quasi-futures" contracts were constructed for each good for delivery one month later.

The analysis of the composition of the portfolios indicated that the bills served as a better hedge against commodity price inflation than did common stocks. It was shown in Table IV-7 that the portfolios were largely composed of Treasury bills with a short position in stocks frequently observed. These results were similar to those of Fama and Schwert (1977a),who found that bills served as a complete hedge against expected inflation while the returns on common stock were negatively related to the expected component of the inflation rate.

Section V. A presented tests of hypotheses concerning the actual success of the "quasi-futures" contracts as hedging instruments. The first test was to indicate if the subsequently observed value at delivery of each "quasi-futures" contract was identical to the observed value of the good. Although the hedge was not perfect, the portfolios did serve as statistically unbiased hedges against commodity price inflation as seen in Tables V-1 and V-2. The second test indicated that the risk properties of the stock-bill portfolios were similar to that of the goods. If each series of "quasi-futures" contracts had the same covariance with the vector of prices $(\tilde{V}_{M1}, \tilde{\Pi}_1, \tilde{B}_1)$ as did each good, then any observed difference between the value of the portfolio and the good each month should be non-systematically related to $(\tilde{V}_{M1}, \tilde{\Pi}_1, \tilde{B}_1)$. The results in Tables V-3 through V-12 showed that these differences were insignificant, indicating the covariance properties of the hedging portfolios and the goods were similar.

The tests were repeated for the period August 1971 through December 1974 when the Nixon wage and price controls were in effect. The results presented in Tables V-13 and V-14, and Tables V-15 through V-24 indicated that the controls had little or no effect on the success of the portfolios as hedging instruments.

An attempt was made to reduce the costs of maintaining the proper portfolio weights by reducing the frequency of the rebalancing. The effect on the quality of each hedge was examined when the portfolios were rebalanced at quarterly and semiannual intervals. A comparison of the observed values of the portfolios and the goods for the quarterly rebalancing case was presented in Table V-25. There was a significant difference in the two values for two of the goods. More importantly,

all of the regressions exhibited positive serial correlation of the residuals as indicated by the low Durbin-Watson statistics. The "quasi-futures" contracts did not provide an unbiased hedge during the unrebalanced months. In addition, rebalancing would have incorporated any information from the previous month's error. The results for the semiannual case presented in Table V-26 indicated a further deterioration in the success of the portfolios as hedging instruments. Thus, rebalancing at quarterly or semiannual intervals in order to avoid costly brokerage commissions is a poor strategy for an investor who wishes to maintain an unbiased hedge. The realization of events each month appears to significantly influence the portfolio weights.

V. B Suggestions for Further Research

This dissertation constructed portfolios which hedged against various consumption bundles used in compiling the Consumer Price Index. A logical step would next be to construct hedging portfolios using actual commodities for which organized commodity exchanges exist. An investor would then be able to compare the price of a "quasi-futures" contract for a particular commodity with the actual price of a futures contract for that commodity. If the price of the "quasi-futures" contract was lower, then an investor could arbitrage the difference in prices by selling the futures contract and purchasing the "quasi-futures" contract. At the delivery date the investor would use the funds from the "quasi-futures" contracts to settle his short position in the futures contract.

An alternate approach for hedging against commodity price inflation might be taken from Manaster (1979). Manaster shows how all nominal efficient portfolios can be made real efficient by the simple addition

of the same hedge portfolio. This hedge portfolio has zero expected return and also requires no net investment. Investors who are concerned with their real wealth can make a costless acquisition of this hedge portfolio, thereby reducing their real risk substantially. The amount of this risk reduction equals the covariance of the nominal return on the hedge portfolio with the rate of inflation.

To implement Manaster's work, one would determine the stock portfolio weights by minimizing the variance of his portfolio for the level of expected price change of the selected good. The hedge portfolio, based on the rate of inflation of the desired good, is then purchased and added to the investor's portfolio.

The success of this strategy as a hedge against commodity price inflation could be compared with the technique suggested in this dissertation. On the surface, the technique used in this dissertation is more appealing. Not only are the investor's tastes for covariances between goods and common stock incorporated, but the covariances among goods and the covariances between goods and Treasury bills are included.

A drawback of using monthly "quasi-futures" contracts for hedging purposes is the frequent and costly rebalancing required to maintain an unbiased hedge. It is suggested that hedge portfolios of longer duration be constructed, followed by testing of their price and covariance properties. Also, using a limited number of bill maturities may be effective when constructing "quasi-futures" contracts in order to reduce the effects of multicollinearity on the regression coefficients. For example, a one month bill and a six month bill may be sufficient. The six month bill would be used in the multiple regressions in order to

attain the correct covariance properties of the portfolios. The one month bill would still be used to adjust the expected value of the portfolios to that of the goods.

APPENDIX A
CREATING A "QUASI-FUTURES" CONTRACT: AN EXAMPLE

Assume an economy containing the shares of two firms, one consumption good, and two long-term bills. In order to create a "quasi-futures" contract for the one consumption good, the first step is to perform the following multiple regression for each of the two firms:

$$\tilde{V}_j = \alpha + \beta_{jM}\tilde{V}_{M1} + \xi_{ji}\tilde{\Pi}_{11} + \delta_{j2}\tilde{B}_{12} + \delta_{j3}\tilde{B}_{13} + \tilde{\varepsilon}_j, \quad j=1,2.$$

Recalling that $b_1{}'\Omega = e_2{}'$ for k=1, we can write

$$[X_{10} X_{20} Y_{02} Y_{03}] \begin{bmatrix} \beta_{1M} & \xi_{11} & \delta_{12} & \delta_{13} \\ \beta_{2M} & \xi_{21} & \delta_{22} & \delta_{23} \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} = [0\ 1\ 0\ 0\ ].$$

Therefore, the quantities of shares of each firm (X_{10} and X_{20}) and the number of each of the long-term bills (Y_{02} and Y_{03}) to hold at time 0 will be

$$X_{10} = \frac{-\beta_{2M}}{\beta_{1M}\ \xi_{21} - \beta_{2M}\ \xi_{11}}$$

$$X_{20} = \frac{\beta_{1M}}{\beta_{1M}\ \xi_{21} - \beta_{2M}\ \xi_{11}}$$

$$Y_{02} = \frac{\beta_{2M}\ \delta_{12} - \beta_{1M}\ \delta_{22}}{\beta_{1M}\ \xi_{21} - \beta_{2M}\ \xi_{11}}$$

$$Y_{03} = \frac{\beta_{2M}\ \delta_{13} - \beta_{1M}\ \delta_{23}}{\beta_{1M}\ \xi_{21} - \beta_{2M}\ \xi_{11}}$$

The final step in creating the "quasi-futures" contract is to add to the portfolio an amount of short-term bills of the quantity

$$Y_{011} = B_{01}\ \{\bar{\Pi}_{11} - [X_{10}\ X_{20}\ Y_{02}\ Y_{03}]\ [\bar{P}_{11}\ \bar{P}_{21}\ \bar{B}_{12}\ \bar{B}_{13}]'\}.$$

APPENDIX B
A GENERAL SOLUTION FOR THE PORTFOLIO WEIGHTS

As previously discussed, the vector b_k', which contains the unit quantities of the respective shares of common stock and long-term bills in the "quasi-futures" contract for good k is solved for using equation (III-7). That is,

$$b_k' = e_{1+k}' \, \Omega^{-1} \tag{III-7}$$

or equivalently

$$b_k' = e_{1+k}' \left[\begin{array}{cc|c} \beta & \Xi & \Delta \\ \hline \underline{0} & \underline{Q} & I \end{array} \right]^{-1} \tag{III-7a}$$

It is possible to find the inverse of matrix Ω in its partitioned form using the methods discussed in Kmenta (1971, p. 612).

$$\text{Let } A = \begin{bmatrix} P_{11} & P_{12} \\ P_{21} & P_{22} \end{bmatrix},$$

$$\text{then } A^{-1} = \begin{bmatrix} P_{11}^{-1} + P_{11}^{-1} P_{12} Q_{22}^{-1} P_{21} P_{11}^{-1} & -P_{11}^{-1} P_{12} Q_{22}^{-1} \\ -Q_{22}^{-1} P_{21} P_{11}^{-1} & Q_{22}^{-1} \end{bmatrix}$$

where $Q_{22} = P_{22} - P_{21}^{-1} P_{11}^{-1} P_{12}$.

For our problem, we can rewrite (III-7a) as

$$b_k' = e_{1+k}' \begin{bmatrix} \underset{N\times N}{(\beta \quad \Xi)^{-1}} & -\underset{N X(T-1)}{(\beta \quad \Xi)^{-1}} \quad \Delta \\ \underset{(T-1)XN}{0} & \underset{(T-1)X(T-1)}{I} \end{bmatrix} \qquad \text{(III-7b)}$$

Since b_k' is a $[1x(N+T-1)]$ vector, the first N elements of b_k' will represent the share weights or the unit quantities of each of the respective shares in the "quasi-futures" contract for good k. Thus, these first N elements of b_k' will only be dependent on the elements of $(\beta\ \Xi)^{-1}$ which does not contain any of the regression coefficients concerning long-term bills. However, the elements of β and Ξ are influenced by the presence of the long-term bills in equation (II-4).

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BIOGRAPHICAL SKETCH

Gerald Douglas Gay was born on October 30, 1952, in Chatham, Ontario, Canada. He graduated from Ocala High School in Ocala, Florida, in 1969. In 1975, he received his Bachelor of Science and Master of Science in Industrial and Systems Engineering from the University of South Florida in Tampa. He is a member of the Tau Beta Pi and Alpha Iota Delta honorary societies.

I certify that I have read this study and that in my opinion it conforms to acceptable standards of scholarly presentation and is fully adequate, in scope and quality, as a dissertation for the degree of Doctor of Philosophy.

Richard H. Pettway, Chairman
Professor of Finance, Insurance, and Real Estate

I certify that I have read this study and that in my opinion it conforms to acceptable standards of scholarly presentation and is fully adequate, in scope and quality, as a dissertation for the degree of Doctor of Philosophy.

Steven Manaster
Assistant Professor of Finance, Insurance, and Real Estate

I certify that I have read this study and that in my opinion it conforms to acceptable standards of scholarly presentation and is fully adequate, in scope and quality, as a dissertation for the degree of Doctor of Philosophy.

Robert W. Kolb
Assistant Professor of Finance, Insurance, and Real Estate

I certify that I have read this study and that in my opinion it conforms to acceptable standards of scholarly presentation and is fully adequate, in scope and quality, as a dissertation for the degree of Doctor of Philosophy.

David A. Denslow, Jr.
Associate Professor of Economics

I certify that I have read this study and that in my opinion it conforms to acceptable standards of scholarly presentation and is fully adequate, in scope and quality, as a dissertation for the degree of Doctor of Philosophy.

H. Russell Fogler

H. Russell Fogler
Professor of Management

This dissertation was submitted to the Graduate Faculty of the Department of Finance, Insurance, and Real Estate in the College of Business Administration and to the Graduate Council, and was accepted as partial fulfillment of the requirements for the degree of Doctor of Philosophy.

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Dean, Graduate School

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